Measuring Tail Risk*

Maik Dierkes[†], Fabian Hollstein[†], Marcel Prokopczuk[†,‡], and Christoph Matthias Würsig[†]

August 5, 2021

ABSTRACT

In this study we comprehensively investigate the usefulness of the tail risk measures proposed in the literature. We evaluate the tail risk measures on the basis of their statistical and economic validity. Our main conclusion is that the option-implied measure of Bollerslev and Todorov (2011b) outperforms all others. It performs well for all tests and can predict not only the occurrence but also the size of future crash events. In addition, the measure is priced in the market: it predicts returns both in the time-series and in the cross-section. Finally, it also has an impact on real economic activity.

JEL classification: G12, C58, G17, G10

Keywords: Tail Risk, Return Forecasting, Tail Event Forecasting

*We thank Torben Andersen, Victor Todorov, and seminar participants at the SOFIE Summer School and Leibniz University Hannover for helpful comments and suggestions. Contact: maik.dierkes@finance.uni-hannover.de (M. Dierkes), hollstein@fcm.uni-hannover.de (F. Hollstein), prokopczuk@fcm.uni-hannover.de (M. Prokopczuk), and wuersig@fcm.uni-hannover.de (C. Würsig).

[†]School of Economics and Management, Gottfried Wilhelm Leibniz University of Hannover, Koenigsworther Platz 1, 30167 Hannover, Germany.

[‡]ICMA Centre, Henley Business School, University of Reading, Reading RG6 6BA, UK.

I. Introduction

Tail risk can be defined as the risk of ending up in an exceptionally bad state of the world. That is, one in which a low-probability, high-impact, i.e., high-marginal-utility event occurs. In asset pricing, such a (left-)tail event is typically associated with high (extreme) negative market returns. Several anecdotal and empirical observations suggest that investors are concerned with such tail risk. First, previous studies find that the prices of out-of-the-money put options, instruments that provide a positive payoff in case of a left tail event, are substantially higher than suggested by theory (Jackwerth, 2000; Bondarenko, 2014). Thus, investors seem to be willing to pay more than advocated by standard models to receive crash insurance. Second, The Economist describes "low-probability, high-impact events" as "a fact of life".[1] Investment practitioners and politicians worry about "fail[ure] to capture [...] the extreme negative tail" (Alan Greenspan) and see one of their main objectives to "remove [...] tail risks, and the perception of tail risks" (Olivier Blanchard).[2,3]

The apparent interest of investors in tail events has sparked a large literature on different tail risk measures. Such measures come in a variety of fashions from highly parameterized models to non-parametric approaches. The underlying data vary from option prices, over historical index and stock returns, to macroeconomic time series. Some measures capture tail risk under the physical, while others rely on the risk-neutral probability distribution. In short, both investors and politicians face a difficult choice between different measures with potentially conflicting predictions.

In this paper, we seek to provide some guidance as to how best to measure tail risk. Our main contribution is a systematic, coherent, and comprehensive evaluation of the tail risk measures proposed in the literature. Knowing how to measure tail risk is very important for academics, investment practitioners, and politicians. Decisions based on an inaccurate measure could lead to vast investment and welfare losses. Furthermore, under the assumption that tail risk is a relevant risk factor, for academics and investors it is essential to accurately ascribe portfolio performance

[1]Lead article "The next catastrophe" in the Economist Issue June 25th 2020.

[2]The first quote is from a speech of Alan Greenspan in 1999: `https://www.federalreserve.gov/boarddocs/speeches/1999/19991014.htm`. The second is from an interview with Olivier Blanchard, then chief economist at the IMF, for The Economist, January 31, 2009.

[3]In addition, the Chicago Board Options Exchange (CBOE) introduced the VIX Tail Hedge Index (VXTH), designed to cope with extreme downward movements in the stock index.

to tail risk exposures. There is thus a great need to identify good tail risk measures.

We analyze a large set of 15 potential tail risk measures. Because they are partially based on very different concepts, theories, assumptions, and underlying data, the different tail risk measures likely measure different things. Indeed, we find that the first two principal components (PCs) of the tail risk measures can only explain 49% of their variation. The correlations between the different measures are moderate at best. In some instances, we even observe negative correlations. Thus, the decision to use a specific measure is non-trivial, with potentially important consequences. The tail risk measures should not be treated as interchangeable.

As a preview, Figure 1 illustrates the vast heterogeneity across the measures. It displays the average levels of the tail risk measures (each standardized to have a mean of zero and standard deviation of one) one day before tail events, as well as one day before placebo (non-tail) events. Some of them have high values (as they should) while others are close to or even below their average before a tail event. Similarly, some measures on average indicate that a tail event is likely to happen when, in fact, no such event is subsequently realized.

After having documented significant heterogeneity between the measures, we continue by defining the desirable criteria a tail risk measure should possess: it should matter both statistically and economically. That is, on the one hand, the tail risk measure should be able to capture both the risk of jumps and deliver an indication about the expected magnitude and quadratic variation caused by tail events. On the other, several studies show that tail risk also matters for investors (e.g., Rietz, 1988; Barro, 2006; Gourio, 2012; Muir, 2017; Dew-Becker, Giglio, and Kelly, 2019). Hence, a tail risk measure should be priced in the market. We thus require a tail risk measure to predict both risk and risk premia.[4]

We devise three main tests. The first two are statistical in nature with (i) a probit predictive regression, predicting two-sigma events and (ii) a prediction of the future left tail variation. With the first test, we examine whether the measures can forecast future tail events, while with the second test we additionally account for the contribution of tail events to the quadratic variation.

[4]Of course, a tail risk measure can also be useful if it only predicts either risk or risk premia. In that case, it could still be used for the tasks it performs well for. Our main goal, however, is to identify measures that can be used for all applications.

The final test (iii) is of an economic nature: we examine whether the measures can forecast future market excess returns.

Our analysis produces a clear winner: The Bollerslev and Todorov (2011b) option-implied left tail measure ($BT11Q$) performs best overall. It works well in forecasting the occurrence of and, in particular, the variation associated with future tail events up to one week ahead. More importantly, it is able to forecast future market excess returns up to one year ahead. $BT11Q$ is among the best measures for each of the individual tasks, and it is the only one that consistently performs well across all tests. On top of that, it is also fairly simple to implement compared to other tail risk measures. It only requires observed deep out-of-the-money put index option prices.

We document that $BT11Q$ can also predict the magnitude, not only the occurrence, of future tail events. Furthermore, it performs well in predicting stock returns in the cross-section. It also predicts real economic activity: $BT11Q$ is a strong negative predictor of the growth of industrial production during the next month. We perform several further tests that underline the robustness of our results. Among others, we show that the results are qualitatively similar across subsample periods, for different multiple regression selection procedures, when predicting the number of jumps, when varying the tail event thresholds, for left tail variation with and without overnight returns, and for different bootstrap approaches to determine the statistical significance. For all tests, the $BT11Q$ measure is amongst the best.

Why does the $BT11Q$ measure perform so well? It appears to combine several desirable properties for a tail risk measure. On the one hand, it uses forward-looking information from options markets. Apart from being forward-looking, options markets have also been shown to contain information about future returns that is not readily found in physical risk measures (Andersen, Fusari, and Todorov, 2015).[5] Most stock-return-based and macroeconomic tail risk measures fail in particular for the return forecasting exercises. In addition, the $BT11Q$ measure has the advantage of being entirely non-paramteric, requiring no estimation of structural parameters. Thereby, it appears to contain substantially less noise than measures which require a parametric optimization

[5] Indeed, David Einhorn refers to the traditional Value-at-Risk (VaR) approach based on historical return data as "an airbag that works all the time, except when you have a car accident" (`https://www.valuewalk.com/wp-content/uploads/2014/05/Grants-Conference-04-08-2008.pdf`).

or which rely on high-frequency or options returns. While this noise does not seem to affect the return predictability exercises that strongly ($BT11Q$ still performs substantially better than all other measures for these), it seems to have a large impact on the statistical tests. None of the other option-implied measures performs nearly as good as $BT11Q$ for predicting future tail events and left tail variation.

The literature contains studies that compare different risk measures in several areas. For example, there is a large literature comparing the ability of different approaches to forecast future volatility (e.g., Andersen and Bollerslev, 1998; Hansen and Lunde, 2005; Jiang and Tian, 2005; Brownlees and Gallo, 2010). There are also studies concerned with how to best forecast covariances (e.g., Symitsi, Symeonidis, Kourtis, and Markellos, 2018) and beta (e.g., Faff, Hillier, and Hillier, 2000; Hollstein and Prokopczuk, 2016; Hollstein, Prokopczuk, and Wese Simen, 2019). Surprisingly, however, to the best of our knowledge, to date no such study exists about tail risk. Given the plethora of different measures that have been proposed over the last decade, we feel there is an urgent need for such a study. Our main contributions are, thus, to (i) define the criteria a good tail risk measure should fulfill and (ii) comprehensively analyze the measures proposed in previous studies based on these criteria. Importantly, we use the same methodology to analyze and evaluate all measures.

The remainder of the paper is organized as follows: In Section II, we present the tail risk measures considered. Section III outlines our evaluation methodology and the data employed. In Section IV, we present the results of our main analysis and in Section V we perform further tests and analyze the robustness of our results. Section VI concludes.

II. Tail Risk Measures

Our aim is to analyze the most comprehensive set of tail risk measures possible. The measure selection is based on two main criteria: (i) relevance/importance and (ii) (public) availability of the underlying data on the measure. Based on these criteria, we have compiled the ensuing list.[6]

[6]Further relevant measures include Andersen et al. (2015); Andersen, Fusari, and Todorov (2017), Agarwal, Ruenzi, and Weigert (2017), Seo and Wachter (2018) and Weller (2018). We refrain from using the measure of

In the following, we introduce the main tail risk measures analyzed in this study. To keep the paper focused, in this section we describe only the main mechanisms of the different measures. *The technical details are in Section OA1 of the Online Appendix.* We categorize the measures into four main groups, mainly based on their underlying data: (i) option-implied measures, (ii) stock-return-based measures, (iii) option-return-based measures, and (iv) tail risk measures based on macroeconomic data.

In Table I, we summarize the measure acronyms and provide brief descriptions, further information about how the different measures can be interpreted, as well as the estimation frequency. Whenever possible, we define the tail risk measure acronyms in accordance with those in the original studies. For cases in which this would lead to names that could not be uniquely identified, we rely on bibliographic information about the study to generate generic acronyms based on the author names, years, and the probability measure under which they are estimated. All measures are estimated such that an investor could have observed these in real time. Thus, whenever estimation of parameters is necessary, it is based on data available at the time.

A. *Option-Implied Measures*

$\boldsymbol{BT11Q}$ (Bollerslev and Todorov, 2011b) is a left tail measure under the risk-neutral probability distribution, which is based on the theoretical framework developed in Bollerslev and Todorov (2011a). Using close-to-maturity deep out-of-the-money put options with constant moneyness, the authors approximate the tail behavior. Intuitively, it is based on the idea that the options will not end up in-the-money at expiration unless a tail event occurs. This results in an expected-shortfall-like measure. We rely on the approximation of Bollerslev and Todorov (2011b).

$\boldsymbol{BT14Q}$ and $\boldsymbol{BTX15Q}$ (Bollerslev and Todorov, 2014; Bollerslev, Todorov, and Xu, 2015) are extensions of the $BT11Q$ left tail measure. For $BT14Q$, the shape of the tail is allowed to be time-varying. Furthermore, instead of using an approximation, we rely on the fully parameterized

Andersen et al. (2015) because the model is highly parameterized, making the estimation computationally very intensive. For Andersen et al. (2017) the weekly options are only available for a limited time period starting in 2011, making a meaningful empirical evaluation infeasible. Finally, we do not have access to the data underlying the measures in Agarwal et al. (2017), Seo and Wachter (2018), and Weller (2018).

model, pooling the options and re-estimating parameters on a weekly basis. For $BT14Q$, Bollerslev and Todorov (2014) only impose a structure on the jump intensity, not on the level shift, smoothing the shift parameter estimates. $\boldsymbol{BTX15prob}$ (Bollerslev et al., 2015) is defined as the probability of a daily loss of 10% or more. The $BTX15Q$ and $BTX15prob$ measures are based on the non-parametric estimation of Lin and Todorov (2019), using the median level and shift parameters computed from different options.[7] Both $BTX15Q$ and $BTX15prob$ are estimated daily.

$\boldsymbol{H\_MRI}$ (Gormsen and Jensen, 2020) is a measure of higher-moment risk. It is defined as the first principal component (PC) of the four moments: skewness, kurtosis, hyperskewness, and hyperkurtosis. The moments are calculated using out-of-the-money put options and the inference techniques of Breeden and Litzenberger (1978) and Martin (2017). To obtain a constant time to maturity, Gormsen and Jensen (2020) interpolate between times to maturity.[8]

$\boldsymbol{RIX}$ (Gao, Lu, and Song, 2019; Gao, Gao, and Song, 2018) is a left tail volatility index. The measure is constructed as the difference between a downside volatility index that, compared to the construction of the VIX, overweights deep out-of-the money put prices and a downside VIX.

$\boldsymbol{TLM}$ (Vilkov and Xiao, 2015) is a parameterized expected shortfall measure. To infer the tail parameters, the authors optimize over the difference between the theoretical (using Extreme Value Theory, EVT) and observed prices of deep out-of-the-money put options. The resulting density can be used to calculate the expected shortfall.

B. Stock-Return-Based Measures

$\boldsymbol{BT11P}$ (Bollerslev and Todorov, 2011b) is a left tail measure under the objective probability measure. Based on intraday high-frequency returns that exceed a certain threshold, the authors estimate the shape and the level of the tail, while adjusting for the time-of-day factor that accounts for intraday variation. Finally, Bollerslev and Todorov (2011b) estimate the tail risk factor based on a time-varying cutoff value.[9]

[7] See also the approach used for the website `https://tailindex.com/` created by Andersen, Todorov, and Fusari.
[8] The authors also show that the first PC loads positively on the kurtosis measures and negatively on the skewness measures. The measure is negatively correlated with volatility. Thus, it tends to be low during volatile periods.
[9] See Section OA1.B of the Online Appendix for further details.

CJI (Christoffersen, Jacobs, and Ornthanalai, 2012) is the jump intensity from a parametric dynamic volatility model with separate dynamic jumps (DVSDJ). The model is estimated with daily return data. To obtain the unobservable measures, Christoffersen et al. (2012) use a filtering technique. We estimate the model using an expanding window and annual reestimation of the coefficients.[10]

JumpRisk and ***JumpRP*** (Maheu, McCurdy, and Zhao, 2013) are the conditional jump intensity and the conditional equity premium due to jumps, respectively. Both measures are derived from a parametric Generalized Autoregressive Conditional Heteroskedastic (GARCH)-jump mixture model. The jump risk premium is calculated as the first derivative of the equity risk premium with respect to the jump intensity. Since they argue that risk premia in the model behave oppositely to the current state of volatility and jump risk, we define *JumpRP* as the inverse of the corresponding Maheu et al. (2013) measure. Like for the Christoffersen et al. (2012) model, we also use an expanding window with annual coefficient reestimation.

$\boldsymbol{\lambda_{Hill}}$ (Kelly and Jiang, 2014) is an expected-shortfall-like measure, derived from the cross-sectional distribution of individual stock returns. The tail threshold is defined as the fifth percentile of all daily unsystematic returns in the cross-section during the past month. The measure is computed using the Hill (1975) power law estimator. Unsystematic returns are defined as the residuals from a regression of the excess returns on the common return factors of Fama and French (1993).

C. Option-Return-Based Measures

ADBear (Lu and Murray, 2019) is the excess return of a bear spread portfolio of S&P 500 options. The bear spread portfolio is designed to pay \$1 if the S&P 500's excess return is below a threshold K_2. To generate this payoff, they go long a put option with strike price K_1 and short a put option with strike K_2, with $K_1 > K_2$, and scale by $K_1 - K_2$. The resulting portfolio pays off \$0 above K_1 and \$1 below K_2. They set K_2 and K_1 to be 1.5 and 1 standard deviations below

[10]The estimated coefficients are then used to calculate the observations for next years jump intensity over the next year. This procedure ensures that the measure is entirely out-of sample, and thus comparable to the other measures used in this study.

zero, respectively, and hold the portfolio for five days.

JUMP (Cremers, Halling, and Weinbaum, 2015) is the return of a vega-neutral and gamma-positive portfolio created from market-neutral straddles written on the S&P 500. We use the daily returns resulting from a strategy with daily rebalancing.

D. Macroeconomic Measures

LE (Adrian, Boyarchenko, and Giannone, 2019) is a measure of the left entropy of the expected gross domestic product (GDP) growth distribution. The authors model the conditional GDP growth distribution using interpolated quantile regressions with the the National Financial Conditions Index (NFCI) as the explanatory variable.

III. Data and Methodology

A. Data

Previous studies rely on several different data sources for estimating tail risk. Following the characterization performed in the previous section, we obtain options as well as stock return data from various sources. First, we obtain data on S&P 500 option prices as well as the corresponding Greeks and the risk-free interest rate and dividend yield from OptionMetrics. To clean the options data, we follow the steps outlined in Carr and Wu (2003, 2009). First, we remove strike prices that are duplicated per day, retaining the one with higher open interest. Second, the bid prices are required to be strictly positive and ask prices cannot be lower than bid prices. Some measures impose a cutoff level for short-maturity options. To be consistent we follow Carr and Wu (2003, 2009) and choose 8 days.

Second, we use the 1-minute prices of the S&P 500 from Thomson Reuters Tick History (TRTH). We follow the steps advocated by Barndorff-Nielsen, Hansen, Lunde, and Shephard (2009) to clean the data. First, we use only data with a time stamp falling during the exchange trading hours, i.e., between 9:30 AM and 4:00 PM EST. Second, we remove recording errors in prices. To be more specific, we filter out prices that differ by more than 10 mean absolute deviations from a rolling centered median of 50 observations. Afterwards, we use the nearest previous entry to assign prices to every 1-minute interval.

Third, we obtain prices of all stocks traded on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the National Association of Securities Dealers Automated Quotations (NASDAQ) that are classified as ordinary common shares (CRSP share codes 10 or 11) from the Center for Research in Security Prices (CRSP). In addition, we obtain data on the S&P 500 index from the same source. We use the total return on the S&P 500 as the market return, subtracting the 1-month Treasury Bill rate from Kenneth French to obtain excess returns.[11]

Finally, we obtain data on the National Financial Conditions Index (NFCI) from the Chicago Federal Reserve and on the GDP from the Bureau of Economic Analysis (BEA). We collect further data from Amit Goyal's webpage (10-year, 3-month, and 1-month Government Bond yields), the St. Louis FRED (AAA and BAA rated corporate bond yields, industrial production), and Martin Lettau's webpage (CAY).[12]

Our sample period extends from 1996 to 2017.[13] Because the aim of this study is, to compare different tail risk measures, we restrict our attention to this period, also for those measures for which data would be available for longer time series.

B. Empirical Test Design

What characterizes a good tail risk measure? Obviously, it should be good at predicting future tail events. To test this property, we devise two statistical tests to gauge the measure's ability to forecast future tail events. Moreover, a good tail risk measure should also matter economically. That is, it should command a risk premium, i.e., be priced by investors (Rietz, 1988; Barro, 2006). To analyze the economic content, we test the measure's ability to forecast future aggregate market returns. In the following sections we describe the corresponding tests in more detail.

[11] The website is `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

[12] Amit Goyal's webpage can be reached as `http://www.hec.unil.ch/agoyal/`. Martin Lettau's webpage is `https://sites.google.com/view/martinlettau/datawebpage`.

[13] The starting date, 1996, is dictated by the fact that both the OptionMetrics and TRTH databases do not start before that date. The ending date of our sample period is restricted by the data availability when we started this project.

B.1. Statistical Tests

The first test we use is a simple forecast analysis of realized tail events. To do so, we use a binary probit model (Vilkov and Xiao, 2015). We define the threshold based on the VIX. The binary dummy variable is defined as follows:

$$D_{t+\Delta t} = \begin{cases} 1 & \text{if } R_{t+\Delta t} \leq -2\sigma_t, \\ 0 & \text{if otherwise,} \end{cases} \tag{1}$$

where $R_{t+\Delta t}$ is the market excess return over the period from t until $t + \Delta t$, with Δt measured in trading days. $\sigma_t = \widehat{VIX}_t/100\sqrt{\Delta t/252}$ is the conditional volatility. VIX_t is the level of the VIX at the end of day t.

To test if the tail risk measure can capture the realization of a 2-sigma tail event, we conduct the following regression:

$$D_{t+\Delta t} = a + b \cdot TRM_t + \epsilon_{t+\Delta t}, \tag{2}$$

where TRM_t is the observation of the tail risk measure at time t.

While the probit model captures the occurrence of tail events, it does not account for by how much the observed returns exceed the specified threshold and how much quadratic variation they account for. Forecasting the quadratic variation due to left tail events might thus be even more important for investors. Hence, for a second test, we examine the measures' abilities to forecast the future realized left tail variation. This measure yields particularly high values if the magnitude of (ex-post) tail realizations are very large (or if there are many tail events over the examined period). Based on Mancini (2001), Bollerslev and Todorov (2011b) propose the following left tail variation measure, which is a special case of the truncated variance:[14]

$$\begin{aligned} LTV_t^{\mathbb{P}} &= \sum_{i=1}^{n-1} r_i^2 \cdot 1_{r_i<(-\alpha_{t,i}\Psi^{0.49})} \\ LTV_{t+\Delta t}^{\mathbb{P}} &= \sum_{i=t}^{t+\Delta t} LTV_i^{\mathbb{P}}, \end{aligned} \tag{3}$$

where r_i denotes an intraday log-return. Following Mancini (2001) and Bollerslev and Todorov (2011b) we include intraday returns only. In Section V.I, we show that the results are qualitatively similar when also including overnight returns in the analysis. Ψ is the length, as a fraction of a day, of each intraday sampling interval. We use market excess returns during $n = 390$ 1-minute intervals every day to estimate Equation (3). $1_{r_i<(-\alpha_{t,i}\Psi_n^{0.49})}$ describes a dummy variable that is 1 if the realized intraday return r_i is below $-\alpha_{t,i}\Psi_n^{0.49}$. $\alpha_{t,i}$ is a time-varying threshold adjusted by a time-of-day (TOD) factor, which accounts for the predictable variation of the intraday returns:

$$\alpha_{t,i} = 4\sqrt{BV_t \wedge RV_t} \cdot TOD_i \cdot \Psi^{0.49}. \tag{4}$$

BV_t and RV_t are the bi-power and realized variation, respectively. To test if the tail risk measure can capture the future left tail variation we run the following regression:

$$LTV_{t+\Delta t}^{\mathbb{P}} = a + b \cdot TRM_t + c \cdot LTV_t^{\mathbb{P}} + d \cdot VIX_t + \epsilon_{t+\Delta t}. \tag{5}$$

We control both for the lagged left tail variation $LTV_t^{\mathbb{P}}$ as well as the current conditional volatility, measured by VIX_t. We do so to see whether the tail risk measures contribute to predicting the left tail variation beyond its own lag and the VIX.

[14]The left tail variation measure is based on a decomposition of the realized variation into continuous and jump variation first proposed by Mancini (2001), which Bollerslev and Todorov (2011b) use to separate the jump variation further into left and right jump variation.

B.2. Economic Tests

For our main economic test, we examine the ability of the different tail risk measures to forecast future market excess returns. If tail risk is a relevant risk-factor in the market, the equity risk premium should include compensation for tail risk. Thus, if tail risk is large, the equity risk premium should be higher than during calm times of low tail risk. Hence, a tail risk measure that is priced in the market should be able to positively forecast future market excess returns.

We use the following regression model to test if the tail risk measures can predict returns:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot Controls_t + \epsilon_{t+\Delta t}. \tag{6}$$

Since there are several variables that have been previously documented to predict future stock returns, we follow Bollerslev, Tauchen, and Zhou (2009) and use several control variables in the vector $Controls_t$: the variance risk premium (VRP), the log dividend price ratio (log(D/P)), the default spread (DFSP), the term spread (TMSP), and the stochastically detrended risk-free rate (RREL).

B.3. Further Methodological Details

Throughout this paper, we report partial rather than "full" R^2s. We do so to emphasize the marginal contribution of each tail risk measure to the explanatory power of a model that may contain several variables.[15] For the probit regressions we calculate the partial R^2 via dominance analysis. We retrieve the average contribution from the dominance analysis following Azen and Budescu (2003). A predictor is dominant if it contributes more to the prediction than another one. We report the measure for general dominance, which is the mean of the average additional contribution on each level. For all other tests we use the partial R^2 of Lindeman, Merenda, and Gold (1980). This measure uses a simple unweighted average of the average contributions of different models of different sizes. It sums up to the unadjusted R^2.

[15] This is particularly important since most of our analyses also contain control variables. In addition, for the analyses with multiple tail risk measures we can gauge the contribution of each individual variable.

For statistical inference, we rely on the wild bootstrap procedure of Rapach, Strauss, and Zhou (2013), which we describe in more detail in the Appendix. The bootstrap preserves the contemporaneous correlation structure in the data, controls for the Stambaugh (1999) bias, and allows or conditional heteroskedasticity in stock returns. To account for autocorrelation, we base all t-statistics in the original and the bootstrap samples on robust Newey and West (1987) standard errors with 29 lags (252 lags for annual horizons). For a robustness test, in Section V.J we also present the results when using a block bootstrap. These are qualitatively similar.

Finally, to reduce the dimensionality in multiple regressions, we follow Bekaert, Harvey, Lundblad, and Siegel (2011) and use the general-to-specific PcGets search algorithm. In multiple steps, this algorithm eliminates insignificant predictor variables. For a robustness test, in Section V.E we alternatively also present the results for the jackknife procedure (Bekaert et al., 2011). We outline both methods in Section B of the Appendix.

IV. Main Analysis

A. Summary Statistics

In Table II, we present the summary statistics of the 15 different tail risk measures. We find that the main characteristics of the measures in our sample match those documented in the literature. The measures are vastly heterogeneous in their means and standard deviations. To account for that and to make the results comparable across measures, we standardize all measures to have a mean of zero and a standard deviation of one for the ensuing tests. Importantly, all but two measures have positive skewness and all measures but one have substantial excess kurtosis. This observation is consistent with the measures' interpretation as capturing the risk of low-probability high-impact events. Once these events become increasingly likely, a tail risk measure should experience a distinct peak. This initial intuition already calls into question the usefulness of those tail risk measures that have negative skewness and/or little to no excess kurtosis, notably λ_{Hill}, $JumpRisk$, and $JumpRP$.

An important feature to distinguish between the different tail risk measures is their persistence. The (daily) first-order autocorrelation exceeds 0.99 for $BTX15prob$, and $JumpRisk$. It is further above 0.90 for $BT11Q$, $BTX15Q$, CJI, H_MRI, and $JumpRP$.[16] The high autocorrelations imply that the tail risk measured by these variables is highly persistent and changes little on a day-by-day basis. On the other hand, there are also two measures with near-zero autocorrelations: $BT11P$ and $JUMP$. The low autocorrelations of these two measures would imply that tail risk changes heavily even over short windows. In part, this is surely caused by large noise in the estimation of these measures. For $JUMP$, the construction of the measure as a daily return likely also plays a role. It appears to be more akin to the first difference in tail risk. The first-order autocorrelations of the remaining measures all exceed 0.60, indicating that according to most measures tail risk is quite persistent.[17] The question, though, whether low, medium, or high persistence is a desirable property of a good tail risk measure is an empirical one, which we seek to answer in this section.

Figures 2, 3, and 4 display the time-series of the standardized tail risk measures. For a better visualization, we further average all daily observations of the tail risk measures during a month. For most measures, we observe distinct peaks during October 2008, the peak of the financial crisis right after the Lehman bankruptcy. In particular, all Bollerslev–Todorov measures show this peak. However, for part of the other measures, we do not observe it. E.g., for λ_{Hill} there is rather a trough than a peak in the time-series at that time. In addition, even among the Bollerslev–Todorov measures we observe substantially different behavior in the time series, with large peaks in some measures that seem to be mostly absent in others. This visual inspection of the tail risk measures suggests that they may not be very strongly correlated among each other and thus contain quite different information.

Table III displays the correlations of the tail risk measures. Consistent with the time-series plots, we find that the correlations are indeed much lower than what one would expect from different measures that are broadly designed to capture essentially the same underlying risk. In particular, the correlation between measures across different groups is typically low.

Among the option-implied measures, we generally observe the highest correlations. E.g., $BT11Q$

[16] The autocorrelation of the λ_{Hill} measure in our sample is somewhat lower than that reported by Kelly and Jiang (2014) (0.75 vs. 0.93). However, this seems to be dependent on the sample period. For their full sample period (1963–2010), we also obtain an autocorrelation of 0.93.

[17] In statistical tests, we use bootstrap procedures (described in the Appendix) to ensure that the inference is robust to this persistence in the explanatory variables.

and $BTX15prob$ have correlations of 0.87 and 0.79 with TLM, respectively. On the other hand, H_MRI is negatively correlated with all but one of the other option-implied measures.[18] Among the stock-return-based measures, the correlations are generally lower. Interestingly, the correlations of $JumpRP$ with most option-implied measures are also relatively high. On the other hand, the correlation of $BT11Q$ with $BT11P$, which are related measures, is relatively low, with 0.37.[19] The correlations of the option-return-based measures with all the others are rather low. Interestingly, the only macroeconomic measure in our dataset, although measured at low-frequency and based on non-financial data, is rather strongly correlated with several of the other measures. E.g., the correlation between LE and TLM is as high as 0.51.

Table III also presents the correlations of the tail risk measures with the VIX, a simple measure of the current conditional volatility. Finding that there is some correlation of tail risk with volatility would be natural. However, the tail risk measures should capture the risk of ending up in particularly bad states of the world on top of "normal" day-to-day variation. We find that many tail risk measures have high correlations with the VIX, e.g., $BT11Q$ (0.89), $BTX15prob$ (0.80), TLM (0.96), and $JumpRP$ (0.85). These high correlations imply that the tail risk measures may allow only little additional insights about tail risk beyond what is captured by the VIX. To account for this, we control for volatility in our empirical tests.

In Table IV we present a principal component (PC) analysis of the tail risk measures. We calculate the first two PCs among all measures, as well as the respective first two PCs within each group of measures. Consistent with our previous results in this section, commonality among the different measures is rather low. The first PC of all measures can only explain 38% of the variation. Together with the second PC, the share rises to only 49%. Thus, it is difficult to capture the information contained in the different tail risk measures with just few PCs.

The largest loadings of the first PC are on $BT11Q$ (0.37), TLM (0.40), and $JumpRP$ (0.35). Thus, these measures appear to be most representative of the common variation in the tail risk

[18] This is consistent with Gormsen and Jensen (2020), who show hat H_MRI tends to be low when volatility is high.

[19] λ_{Hill} has negative correlations with almost all other measures, apart from H_MRI. The latter observation is consistent with Kelly and Jiang (2014), who show that λ_{Hill} loads negatively on skewness and positively on kurtosis, as does (by construction) H_MRI.

measures. Within the subgroups, the degree of commonality is somewhat larger. The first two PCs in each subgroup capture at least 59% of the variation in the tail risk measures. The highest loadings of the first PC among the option-implied measures are on $BT11Q$ (0.44) and TLM (0.48). Among the stock-return-based measures, the highest PC loadings are on $BT11P$ and $JumpRP$. However, being able to capture common variation in the tail risk measures may be a misguided objective for the selection of a certain measure. We should rather judge the measures based on their ability to forecast future tail events and capture risk premia.

B. Statistical Tests

We start with the statistical tests. We use three different forecast horizons: (i) one day ($Daily$), (ii) one week ($Weekly$), and (iii) one month ($Monthly$).[20] We do not look at longer horizons for this analysis because being able to predict realized tail events or variation in the far future appears unrealistic. Beginning with the probit model, we examine how well the tail risk measures perform in forecasting future tail events. For each measure and forecast horizon we conduct separate regressions of the (horizon-specific) dummy variables on the lagged standardized tail risk measures.

First, in Figure A1 of the Online Appendix, we illustrate the timing of realized left tail events. We separately depict these for the daily, weekly, and monthly horizons. There is some clustering of realized left tail events during specific crisis periods such as the burst of the dot-com bubble and the recent financial crisis. Interestingly, we find that not all daily left tail realizations lead to weekly or monthly left tail observations. Similarly, part of the weekly and monthly tail events occur without being driven by single or multiple daily tail observations.

The probit regression results are presented in Table V. At the daily level, we find that many of the tail risk measures have some predictive power for future tail events. The 3 measures that show the highest R^2s and that are statistically significant are, in order, $JumpRP$, CJI, and $BT11Q$. Figures A2 to A4 of the Online Appendix plot the fitted values of the regressions at the daily frequency along with the realized tail events. These fitted values visualize the time-varying

[20]Four of the measures are not available on a daily frequency. In the case of these measures, we constantly extrapolate the last weekly, monthly, or quarterly observation until new information becomes available.

probabilities of a crash implied by the regression model. While many measures are largely useless for predicting future left tail events at the daily horizon, it becomes apparent why $JumpRP$, CJI, and $BT11Q$ perform best. They often yield their most pronounced peak implied tail event probabilities around actual tail event realizations. For all measures, periods in which the models suggest high probabilities of a tail event without one actually occurring do not appear to be all that common.

At the weekly and monthly horizons, the overall performance of the measures becomes much weaker. None of the three tail risk measures that perform best at the daily horizon yields a significant positive predictive coefficient. At the weekly horizon, $ADBear$ yields a weakly significant positive predictive coefficient. At the monthly horizon, $JumpRisk$ and $JUMP$ are able to predict future tail events. No tail risk measure can predict future left tail events for more than one horizon.

It is important to mention that we require the tail risk measures to be positively related to future tail events. That is, a high tail risk measure should be associated with a higher probability of a future tail event. At the monthly horizons, for example, $BTX15prob$ and RIX even yield slope coefficients that are significantly negative. Such results are surely inconsistent with being a good tail risk measure.

Beside the individual tail risk measures, we also repeat the probit regressions with the first PC of all measures and among the different subgroups. We find that the first PC of all measures and that only using stock-return-based measures significantly predict tail events at the daily frequency. At the weekly and monthly horizons none of the PCs significantly predicts future tail events.

We further report the results of multiple probit regressions in Table VI. For each horizon, we select the measures with PcGets. For the daily forecast horizon, the selected measures that have significant positive coefficients are $BT11Q$ and CJI. For the weekly horizon, only $BT14Q$ is selected and yields a significant positive coefficient. At the monthly horizon, we cannot detect any significant positive coefficient.

Next, we move from a left-hand-side variable that only indicates whether there is a tail event or not to one that also includes information about the magnitude of the tail event and, correspondingly, the variation it causes. That is, we predict the realized left tail variation (also standardized to have a mean of zero and a standard deviation of one). We present the results in Table VII. We study the same horizons as before (daily, weekly, and monthly) and control for the lagged left tail variation measure and the VIX.

Starting with the daily frequency, we find that $BT11Q$ turns out to be the best predictor. It yields the largest slope coefficient and highest partial R^2. The slope coefficient of 0.19 indicates that, all else equal, an increase in $BT11Q$ by one standard deviation increases the left tail variation by 0.19 standard deviations. The measures $BT14Q$, $BTX15Q$, H_MRI, $JumpRisk$, and $JUMP$ are also significant positive predictors of future left tail variation at the daily frequency. At the weekly horizon, only $BT11Q$ and $JumpRisk$ yield a significant positive slope coefficient. At the monthly forecast horizon, $BT11P$, $JUMP$, and LE are significant predictors of the future left tail variation.

Turning to the PCs, we find that only the first PC of all measures has predictive power for future left tail variation at all horizons. The first PC of the option-return-based measures further has predictive power at the monthly horizon.

We present the results for the multiple regressions to predict the future left tail variation in Table VIII. $BT11Q$ turns out as the best predictor of realized left tail variation for the daily horizon. It has by far the largest slope coefficient and partial R^2. At the weekly and monthly horizon, on the other hand, LE performs performs best.

Thus, overall, the statistical analysis places $BT11Q$ in pole position in the tail risk measure horse race. It performs well not only for predicting future tail events, but seems to also accurately capture the future left tail variation over short horizons. For predicting tail events and left tail variation over longer horizons, other measures perform well, most notably $BT14Q$ and $JumpRisk$ for tail events and LE for left tail variation.

C. Economic Tests

Finally, we turn to the question of whether tail risk is priced in the market. While part of the tail risk measures are developed for slightly differing purposes, the majority of studies appear to argue that their tail risk measure is priced. Hence, this analysis is equitable. We examine whether the tail risk measures have predictive power for future market excess returns over various horizons. For this analysis, we include an annual forecast period in addition to the daily, weekly, and monthly horizons. We do so for two reasons. First, it is common in the return predictability literature to also consider longer horizons. Second, long-horizon returns can be also be influenced by tail risk expectations, while for the statistical tests we would need to observe actual tail event realizations, which are exceedingly rare at long horizons. In the analysis we are interested in the marginal effect of the tail risk measures, controlling for several other predictor variables (see the details in Section III.B.2). We present the results in Table IX. As in Kelly and Jiang (2014), we use annualized returns in percentage points.

As for the previous analyses, we find that $BT11Q$ again performs very well. It is the only measure that significantly predicts future market excess returns at the daily, weekly, monthly, and annual horizons. For each of the horizons, the size of the predictive coefficient and/or the partial R^2 are among the top 3. At the daily and weekly horizon, the slope coefficient is even the largest among all models. For example, at the daily frequency, a one-standard-deviation increase in $BT11Q$, all else equal, implies that the annualized market excess return increases by 35.96 percentage points. The partial R^2 is 0.52%.

$BTX15prob$, TLM, $BT11P$, and $ADBear$ also have predictive power at the daily horizon, but their impact on market excess returns is somewhat smaller. Out of these, only $BT11P$, and $ADBear$ also have predictive ability at both the weekly and monthly horizons. None of these variables can predict excess returns one year ahead. On the other hand, the predictive power of λ_{Hill} seems to start only at the annual forecast horizon. With a partial R^2 of 9.38%, though, the measure's long-term predictive ability is very strong.[21]

The PCs also perform quite well for predicting future market excess returns. All yield significant coefficients for the daily horizon. Furthermore, all PCs except that from only option-implied measures significantly predict future returns at both the weekly and monthly horizons.

[21] Kelly and Jiang (2014) also report a good performance of λ_{Hill} for the 3- and 5-year forecast horizons in their 1963–2010 sample period.

The results for the multiple return predictions are in Table X. Confirming our previous results, the PcGets selection procedure selects $BT11Q$ for the daily, monthly, and annual horizons. For each of these horizons, the measure yields a statistically significant slope coefficient. $BT11P$ is selected and yields a significant positive slope coefficient at the daily and weekly horizons, making it suitable for predicting returns over short horizons.

V. Further Analyses and Robustness Tests

A. Tail Event Return Predictability

In the main analysis, we have separately analyzed the statistical and economic value of the tail risk measures. Next, we perform a joint analysis that also enables us to analyze whether the size of the tail event is predictable. That is, in the absence of a tail event the tail risk premium should be larger the larger the tail risk. If the tail risk is realized in a sudden market event, on the other hand, the exact opposite relationship should hold: the tail event (negative market excess return) should be larger the higher the previous tail risk.

To analyze these subtleties, we perform an alternative return predictability regression. We use the dummy variable defined in Equation (1) to isolate periods with tail events from those without and run the following regression:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot D_{t+\Delta t} \cdot TRM_t + d \cdot D_{t+\Delta t} + e \cdot Controls_t + \epsilon_{t+\Delta t}. \qquad (7)$$

As before, we expect a good tail risk measure to have a positive b coefficient. The c coefficient on the interaction of the tail risk measure with the dummy variable, on the other hand, should be very low. To understand that, remember from Equation (1) that the dummy variable $D_{t+\Delta t}$ indicates that we are in a tail state. Thus, the higher the level of the tail risk measure, the lower should be the future (negative) realized tail-state return. Hence, with this specification we essentially jointly analyze both the tail risk measures' risk premia and whether they can predict the size of future tail events.

We present the results in Table XI.[22] We find that $BT11Q$ also performs well for all horizons in this more granular analysis. For all forecast horizons except monthly, the b coefficient is significantly positive and the c coefficient is significant and negative, as it should be. $BTX15prob$, TLM, $BT11P$, $JumpRP$, $ADBear$, and the first PCs also work well.

In Table XII, we present the corresponding multiple regression analysis. Consistent with the previous results, $BT11Q$ performs well. The measure yields the largest b coefficients at the daily and monthly horizons. In addition, the c coefficient at the weekly horizons is significantly negative with the highest partial R^2. Other measures' b and c coefficients are sometimes selected and yield more significant results, but none of them consistently performs similarly well to $BT11Q$.

B. *Tail Risk and the Cross-Section of Stock Returns*

Next, we analyze the impact of the tail risk measures on the cross-section of stock returns. That is, we conduct a cross-sectional return prediction test to analyze whether stocks with higher tail risk loadings exhibit larger expected returns.

For this analysis, as Kelly and Jiang (2014), we use the same design as for the predictive regressions. We first estimate the stocks' sensitivities to tail risk using a rolling historical window. We use a window length of one month for all measures available at the daily frequency.[23] At the end of each month, the factor loadings are then estimated by the following predictive regression:

$$R^i_{t+\Delta t} = a^i + b^i \cdot TRM_t + \epsilon^i_t, \tag{8}$$

where $R^i_{t+\Delta t}$ denotes the excess return of stock i during the period t until $t + \Delta t$. We focus on a Δt of 1 day. TRM_t is the tail risk measure at time t. We sort the stock based on the estimated b^i and hold the portfolio for one month. Afterwards, we repeat the entire procedure.

Stocks that perform comparably better following high-tail-risk observations are very desirable for investors. These stocks essentially insure high-marginal-utility states. Thus, investors likely

[22] Note that, as before, we skip the annual horizon due to lack of sufficient observations for the tail dummy variable.

[23] For all other frequencies, the rolling window length is defined based on a mechanical rule: we require at least 22 non-overlapping observations. Thus, the window for weekly, monthly, and quarterly variables is six months, two years, and six years, respectively.

have a strong demand for those stocks that yield high b^is in Equation (8). This increased demand leads to high prices and, hence, low unconditional average returns. These low average returns are akin to an insurance premium paid by investors. On the other hand, those stocks that perform poorly following an observation of high tail risk are undesirable for investors. Hence, they have to pay a higher return in order to induce investors to hold them.

We present the results for value-weighted portfolios in Table XIII. We find that for $BT11Q$ the portfolio with the lowest tail risk loadings has an average annualized excess return of 11.01%. Thus, stocks that perform poorly after a high-tail-risk observation have high returns. On the other hand, the stocks in the portfolio with the highest tail risk loadings only yield an average annualized excess return of only 1.53%. Thus, stocks that do well following a high-tail-risk observation perform less well on average. The difference between the high and low portfolios is −9.48% per year on average. These results are consistent with the intuition described above. Stocks that do well following the observation of high tail risk appear to be very desirable for investors and trade at a premium.

While the results for $BT11Q$ are clear and consistent with economic theory, we find that the vast majority of the other tail risk measures do not yield significant negative high–low portfolio excess returns. Thus, these measures do not seem to be priced in the cross-section of stock returns. Exceptions include $BTX15prob$, TLM, $BT11P$, and $JumpRisk$. The cross-sectional tail risk premia implied by these measures, however, are substantially smaller than that of $BT11Q$.

In Table A1 of the Online Appendix, we also present the results for equally weighted portfolios. These are qualitatively similar. Finally, in Table A2 of the Online Appendix, we report the value-weighted Fama and French (2015) five-factor model alphas instead of raw excess returns. These are also qualitatively very similar. Thus, the pricing of tail risk appears to be distinct from that of market risk as well as the other factors in this model.

C. Tail Risk and Real Economic Activity

Facing high tail risk, new investments in the real economy may be delayed and hiring of new staff paused (Kelly and Jiang, 2014; Gormsen and Jensen, 2020). Thus, if tail risk affects real economic activity, it should have an impact on growth in industrial production. Therefore, we also run predictive regressions of log industrial production growth on the different tail risk measures. We use the following regression model:

$$IND_{t+\Delta t} = a + b \cdot \overline{TRM}_t + \epsilon_{t+\Delta t}, \tag{9}$$

where $IND_{t+\Delta t}$ is the log change in industrial production over the period Δt. Since industrial production is only available on a monthly level, we focus on monthly and annual prediction windows. Therefore, $\overline{TRM}_t$ is the current observation of a tail risk measure, computed as the average of all observations during month t.

We present the results in Table XIV. Indeed, we find that tail risk has an impact on the growth in industrial production. For example, at the monthly frequency, a one-standard-deviation increase in $BT11Q$ decreases the log industrial production growth by 0.24 percentage points. The economic impact is the largest among all tail risk measures. At the annual frequency, only $BT11Q$, $BT11P$, and $ADBear$ are significant negative predictors of future industrial production growth.

D. Subsample Analysis

Next, we analyze the robustness of the tail risk measures' return predictability for two distinct subsamples. For that purpose, we divide our total sample period in two roughly equal halves: one ending in 2007, before the Financial Crisis, and the other starting from 2008 until the end of our sample period. The extreme returns around the peak of the Financial Crisis may be influential and drive part of the overall predictability results. By running the main economic test separately for both subsamples, we can therefore assess how stable the predictability is.

The results for the pre-2008 period are in Tables A3 and A4 of the Online Appendix. We find that $BT11Q$ significantly predicts returns at the daily, weekly, and monthly horizons. Other measures that perform well include $BT11P$, $ADBear$, and $JUMP$. The model selection algorithm picks $BT11Q$ for three out of four horizons, for which it also yields significantly positive coefficients. Thus, our results for the first half of the sample period are consistent with those for the full period.

Next, we examine the post-2008 period. The corresponding results are in Tables A5 and A6 of the Online Appendix. We find that $BT11Q$ significantly predicts future market excess returns at the daily, weekly, and annual horizons. $BT11P$ and $ADBear$ also perform well for the second half of the sample period. Importantly, the predictive ability of the best measures thus appears to be rather stable over time.

E. Jackknife Model Selection

In a next step, we analyze the robustness of our main results to the model selection algorithm in the multiple regression analysis. That is, instead of the PcGets algorithm, we alternatively employ a jackknife procedure, which we describe in detail in Appendix B.

We present the results in Tables A7, A8, and A9 of the Online Appendix. These are overall qualitatively similar to those for the PcGets selection algorithm. Although the two approaches select different measures in some instances, the big picture remains the same. While $BT11Q$ is not selected for the tail event prediction, it is instead selected for all horizons and yields statistically significant coefficients for predicting left tail variation. For the return predictability, $BT11Q$ also turns out to be the best model under the jackknife selection.

F. The Number of Jumps

We also devise an alternative statistical test to evaluate the tail risk measures: the number of jumps. That is, for each forecast window, we simply count the number of realized jumps (NLJ) based on the jump test implicit in Equation (3).[24] Analogously to the test for the left tail variation, we then perform univariate regressions of the standardized realized number of negative jumps on each lagged tail risk measure:

$$NLJ_{t+\Delta t} = a + b \cdot TRM_t + c \cdot NLJ_t + d \cdot VIX_t + \epsilon_{t+\Delta t},$$

where all variables are as previously defined.

[24] Technically, we estimate Equation (3) without multiplying the jump-imposed dummy variable with the squared returns. Thus, we simply count by summing up ones if there are jumps.

We present the results in Table A11 of the Online Appendix. $BT11Q$ performs well also for this test. For all forecast horizons, it yields the highest slope coefficient, which is also statistically significant in every case.

G. Tail Threshold

In Figures A5 – A7 of the Online Appendix we vary the threshold to define the tail events for the probit regressions. We display the t-statistics of the b coefficient in Equation (2) for tail thresholds varying between -0.2 and -2 times the conditional volatility (in steps of 0.1). The results for common thresholds are qualitatively similar to those of Table V. At the daily forecast horizon, $BT11Q$ and $JumpRP$ can predict future tail events for all analyzed tail thresholds. At the weekly and monthly horizons, the performance is typically more dependent on the tail threshold. Some measures succeed for certain thresholds. On the other hand, part of the measures only perform well for extreme thresholds; e.g., $ADBear$ at the weekly horizon and $JumpRisk$ and $JUMP$ at the monthly horizon.

H. The Impact of Future Tail Events on Tail Risk

Additionally, we investigate a specification that essentially reverses the direction of the probit regression and thereby examines the robustness of Figure 1:

$$TRM_t = a + b \cdot D_{t+\Delta t} + \epsilon_{t+\Delta t}.$$

We display the b coefficient estimates along with their 90% confidence intervals based on robust Newey and West (1987) standard errors in Figures A8 to A10 of the Online Appendix. A good measure should have a positive and statistically significant b coefficient. The results are qualitatively very similar to those of the tail event predictability analysis.

I. Left Tail Variation With Overnight Returns

We also examine the robustness of our results for the predictability of future realized left tail variation to also including overnight returns. We present these results in Tables A12 of the Online Appendix. The results are qualitatively similar to those without including overnight returns. If anything, they are even more favorable for $BT11Q$, which performs best for all forecast horizons.

J. Block Bootstrap

Finally, we examine the robustness of our results to the bootstrap method to determine the statistical inference. For that purpose, we conduct a block-bootstrap. As advised by Lahiri (1999), we use overlapping blocks. The block length is $n^{1/3}$ or the number of overlapping observations, whichever is larger (Hall, Horowitz, and Jing, 1995). The block bootstrap places more emphasis on the dependence structure in the residuals and is a reality check mainly for the long-term predictive performance of the tail risk measures.

We present the results for the predictability of left tail variation in Tables A13 and A14 of the Online Appendix. These are very similar to those for the wild bootstrap. For return predictability, we present the results in Tables A15 and A16 of the Online Appendix. While the long-term return predictability is indeed somewhat more modest, overall the results are also very similar for the return predictability when using a block bootstrap.

VI. Conclusion

We contribute to the literature by conducting a comprehensive empirical analysis of a wide range of tail risk measures that have been proposed over the recent decade. We detect a large heterogeneity across different tail risk measures measures. The first two principal components explain only 49% of their total variation, while some tail risk measures are even negatively correlated. This finding sends a clear warning to researchers and practitioners not to treat different tail risk measures as interchangeable.

We find that the option-implied measure of Bollerslev and Todorov (2011b), $BT11Q$, performs best. Further refinements of the Bollerslev and Todorov (2011b) measures by the same authors appear to be of limited practical value. $BT11Q$ performs well for all tests: It can predict the occurrence and the magnitude of future tail events as well as the variation caused by them. The measure also predicts market excess returns at horizons up to one year. In addition, it is priced in the cross-section of stock returns and affects real economic activity. Other measures only perform well at most for part of the tasks (while most consistently underperform the winning measure).

Appendix A. Wild Bootstrap Procedure

For statistical inference, we generally rely on the multivariate wild bootstrap of Rapach et al. (2013). For example, with the predictive regression in Equation (6), the wild bootstrap procedure retains the residuals from the main estimation procedure and the residuals of a $VAR(1)$ from all RHS variables, the parameters are estimated with a reduced-bias VAR estimate by iterating on the Nicholls and Pope (1988) expression for the analytical bias of the OLS estimates. The coefficients and residuals of these estimations are used to build pseudo-samples for all RHS variables. In each pseudo-sample, the LHS returns are constructed under the null of no predictability. The RHS variables in each pseudo-sample rely on the reduced-bias $VAR(1)$ parameter estimates from the original residuals, multiplied with standard normal random variables. This procedure preserves the contemporaneous correlation of the variables and captures conditional heteroskedasticity. Using the pseudo-samples, one can calculate the t-statistics for the usual regression. With this distribution of t-statistics, one can obtain the p-values based on the location of the sample t-statistic in this distribution.

For example, in a return predictability regression, assuming we have only one control variable (to keep the notation short we only use the VRP; the extension to multiple control variables is straightforward), we estimate the following set of regressions:

$$R_{t+\Delta t} = a + b \cdot TRM_t + VRP_t + \epsilon_{t+\Delta t}$$

$$TRM_{t+1} = \rho_{d,0} + \rho_{d,1} TRM_t + \rho_{d,2} VRP_t + \nu_{d,t+1}$$

$$VRP_{t+1} = \rho_{b,0} + \rho_{b,1} VRP_t + \rho_{b,2} TRM_t + \nu_{b,t+1}.$$

We retain the estimated coefficients: $(\hat{\rho}_{d,0}, \hat{\rho}_{d,1}, \hat{\rho}_{d,2})$, $(\hat{\rho}_{b,0}, \hat{\rho}_{b,1}, \hat{\rho}_{b,2})$ as well as the residuals $\hat{\epsilon}_{t+\Delta t}$,

$\hat{\nu}_{d,t+1}$ and $\hat{\nu}_{b,t+1}$. Using these estimates and residuals, we build the pseudo-samples under the null:

$$
\begin{aligned}
R^*_{t+\Delta t} &= \bar{R} + \hat{\epsilon}_{t+\Delta t} w_{t+1} \\
TRM^*_{t+1} &= \hat{\rho}_{d,0} + \hat{\rho}_{d,1} TRM^*_t + \hat{\rho}_{d,2} VRP^*_t + \hat{\nu}_{d,t+1} w_{t+1} \\
VRP^*_{t+1} &= \hat{\rho}_{b,0} + \hat{\rho}_{b,1} VRP^*_t + \hat{\rho}_{b,2} TRM^*_t + \hat{\nu}_{b,t+1} w_{t+1},
\end{aligned}
$$

where w_{t+1} is a standard normally distributed variable to produce the pseudo-sample. We repeat this procedure 1,000 times. The p-value represents the percentage of times the t-statistics of the pseudo-sample are greater (for positive coefficients) or smaller (for negative coefficients) than the t-statistic of the original sample. To account for autocorrelation, we base all t-statistics in the original and the bootstrap samples on robust Newey and West (1987) standard errors with 29 lags (252 lags for annual horizons).

Appendix B. Multiple Regression Selection Procedures

PcGets Procedure

For the multiple regression analysis, we use the general-to-specific search algorithm of Hendry (1995) and Hendry and Krolzig (2001). We follow the detailed implementation as described by Bekaert et al. (2011) in their Appendix Table 4. For convenience, we provide the steps here:

1 Estimate a general model (G1) including all variables.

  a If all coefficients are individually significant at a level of 0.025, G1 is the final model (*t-test*).

  b If an F-test cannot reject the null hypothesis at a level of 0.500 that all coefficients are zero, or all coefficients but the constant are zero, the null not rejected constitutes the final model (F-*test*).

2 Pre-search tests

  a Top-down tests: We test an expanding list of coefficients (from smallest to largest t-statistic). If an F-test does not reject the null hypothesis at a level of 0.500 when we add a coefficient, we remove the corresponding explanatory variable. The resulting reduced model is the new general model G2 (F-*test*).

b Estimate G2 and repeat (a) with the new model at a level of 0.250 for the null hypothesis (F-*test*).

c Bottom-up tests: We test a decreasing list of coefficients (from largest to smallest t-statistic). If the F-test does not reject at a level of significance of 0.025, remove the additional variables. The reduced model is the new general model (G3) (F-*test*).

3 Multiple-path tests

a Estimate G3. If all coefficient estimates are individually significant at a significance level of 0.025, G3 is the final model (t-*test*).

b Initiate search paths, re-estimate the model after removing all variables with p-values above (0.90, 0.70, 0.50, 0.25, 0.10, 0.05, 0.01, 0.001). This leaves 8 paths. Additionally start a path for each variable that is insignificant at the 0.025 level. Proceed with these paths in (c).

c As long as insignificant estimates survive at a level of 0.025, drop the least significant one and re-estimate (t-*test*). A search path is abandoned if no coefficients are significant. A path arrives at a terminal model if all coefficient estimates are significant.

4 Encompassing

If all search paths are abandoned, G3 is the final model.

If there is only one terminal model, it is the final model.

If there are multiple terminal models, test each model against the union of all models with an F-test with a significance level of 0.025 (F-*test*).

If all models are rejected, the union is the final model.

If only one model is not rejected, it is the final model.

If multiple models are not rejected, they are tested against their union (after removing any rejected models).

If only one model is not rejected, it is the final model.

If all models are rejected, the union is the final model.

If no model is rejected, their union is the new general model (G4).

5 Repeat steps 3 and 4 for the new general model (G4)

If there is only one terminal model, it is the final model.

If there are multiple terminal models, they are tested against their union:

If only one model is not rejected, it is the final model.

If all models are rejected and their union equals G4, then G4 is the final model.

If several models are not rejected and their union does not equal G4, their union is the new general model (G5) and steps 3 and 4 are repeated.

If several models are not rejected and their union equals G4, the model with the smallest Schwarz criterion is the final model.

Jackknife Procedure

Alternatively, we follow Bekaert et al. (2011) and also consider a jackknife procedure. It entails the following steps. First, for each tail risk measure (the candidate variable), we perform a regression with a selection of the other variables and the candidate variable. First, we randomly select the number of variables to be used. We require at least 30% of all variables to be included in the selection procedure, which amounts to 9 variables for the return predictability regression. Second, we randomly select the chosen number of variables from all available variables without replacement. We then run a regression with all these variables. Third, we eliminate all variables with t-statistics whose magnitudes are below one (except for the candidate variable). Then, we run another regression with the remaining variables. In the last step, the coefficient of the candidate variable is retained. This procedure is repeated one thousand times for each candidate variable, calculating 90% confidence intervals. All candidate variables whose confidence intervals exclude zero are retained for the final multiple regression.

REFERENCES

Adrian, Tobias, Nina Boyarchenko, and Domenico Giannone, 2019, Vulnerable Growth, *American Economic Review* 109, 1263–89.

Agarwal, Vikas, Stefan Ruenzi, and Florian Weigert, 2017, Tail Risk in Hedge Funds: A Unique View from Portfolio Holdings, *Journal of Financial Economics* 125, 610–636.

Andersen, Torben G., and Tim Bollerslev, 1998, Answering the Skeptics: Yes, Standard Volatility Models do provide accurate Forecasts, *International Economic Review* 39, 885–905.

Andersen, Torben G., Nicola Fusari, and Viktor Todorov, 2015, The Risk Premia Embedded in Index Options, *Journal of Financial Economics* 117, 558–584.

Andersen, Torben G., Nicola Fusari, and Viktor Todorov, 2017, Short-term Market Risks Implied by Weekly Options, *Journal of Finance* 72, 1335–1386.

Azen, Razia, and David V. Budescu, 2003, The Dominance Analysis Approach for Comparing Predictors in Multiple Regression, *Psychological Methods* 8, 129–148.

Bakshi, Gurdip, Nikunj Kapadia, and Dilip Madan, 2003, Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options, *Review of Financial Studies* 16, 101–143.

Barndorff-Nielsen, Ole E., Peter R. Hansen, Asger Lunde, and Neil Shephard, 2009, Realized Kernels in Practice: Trades and Quotes, *Econometrics Journal* 12, 1–32.

Barndorff-Nielsen, Ole E., and Neil Shephard, 2004, Power and Bipower Variation with Stochastic Volatility and Jumps, *Journal of Financial Econometrics* 2, 1–37.

Barndorff-Nielsen, Ole E., and Neil Shephard, 2006, Econometrics of Testing for Jumps in Financial Economics using Bipower Variation, *Journal of Financial Econometrics* 4, 1–30.

Barro, Robert J., 2006, Rare Disasters and Asset Markets in the Twentieth Century, *Quarterly Journal of Economics* 121, 823–866.

Bekaert, Geert, Campbell R. Harvey, Christian T. Lundblad, and Stephan Siegel, 2011, What Segments Equity Markets?, *Review of Financial Studies* 24, 3841–3890.

Black, Fischer, and Myron Scholes, 1973, The Valuation of Options and Corporate Liabilities, *Journal of Political Economy* 81, 637–654.

Bollerslev, Tim, George Tauchen, and Hao Zhou, 2009, Expected Stock Returns and Variance Risk Premia, *Review of Financial Studies* 22, 4463–4492.

Bollerslev, Tim, and Viktor Todorov, 2011a, Estimation of Jump Tails, *Econometrica* 79, 1727–1783.

Bollerslev, Tim, and Viktor Todorov, 2011b, Tails, Fears, and Risk Premia, *Journal of Finance* 66, 2165–2211.

Bollerslev, Tim, and Viktor Todorov, 2014, Time-varying Jump Tails, *Journal of Econometrics* 183, 168–180.

Bollerslev, Tim, Viktor Todorov, and Lai Xu, 2015, Tail Risk Premia and Return Predictability, *Journal of Financial Economics* 118, 113–134.

Bondarenko, Oleg, 2014, Why are Put Options so Expensive?, *Quarterly Journal of Finance* 4, 1450015.

Brave, Scott, and R Andrew Butters, 2012, Diagnosing the Financial System: Financial Conditions and Financial Stress, *International Journal of Central Banking* 8, 191–239.

Breeden, Douglas T., and Robert H. Litzenberger, 1978, Prices of State-Contingent Claims Implicit in Option Prices, *Journal of Business* 51, 621–651.

Brownlees, Christian T., and Giampiero M. Gallo, 2010, Comparison of Volatility Measures: A Risk Management Perspective, *Journal of Financial Econometrics* 8, 29–56.

Carr, Peter, and Liuren Wu, 2003, What Type of Process Underlies Options? A Simple Robust Test, *Journal of Finance* 58, 2581–2610.

Carr, Peter, and Liuren Wu, 2009, Stock Options and Credit Default Swaps: A Joint Framework for Valuation and Estimation, *Journal of Financial Econometrics* 8, 409–449.

Christoffersen, Peter, Kris Jacobs, and Chayawat Ornthanalai, 2012, Dynamic Jump Intensities and Risk Premiums: Evidence from S&P 500 Returns and Options, *Journal of Financial Economics* 106, 447–472.

Cremers, Martijn, Michael Halling, and David Weinbaum, 2015, Aggregate Jump and Volatility Risk in the Cross-section of Stock Returns, *Journal of Finance* 70, 577–614.

Dew-Becker, Ian, Stefano Giglio, and Bryan Kelly, 2019, Hedging macroeconomic and financial uncertainty and volatility, *Yale ICF Working Paper* .

Faff, Robert W., David Hillier, and Joseph Hillier, 2000, Time Varying Beta Risk: An Analysis of Alternative Modelling Techniques, *Journal of Business Finance & Accounting* 27, 523–554.

Fama, Eugene F., and Kenneth R. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F., and Kenneth R. French, 2015, A Five-Factor Asset Pricing Model, *Journal of financial economics* 116, 1–22.

Gao, George P., Pengjie Gao, and Zhaogang Song, 2018, Do Hedge Funds exploit rare Disaster Concerns?, *Review of Financial Studies* 31, 2650–2692.

Gao, George P., Xiaomeng Lu, and Zhaogang Song, 2019, Tail Risk Concerns Everywhere, *Management Science* 65, 3111–3130.

Gormsen, Niels Joachim, and Christian Skov Jensen, 2020, Higher-moment risk, *University of Chicago Working Paper* .

Gourio, Francois, 2012, Disaster risk and business cycles, *American Economic Review* 102, 2734–66.

Hall, Peter, Joel L. Horowitz, and Bing-Yi Jing, 1995, On Blocking Rules for the Bootstrap with Dependent Data, *Biometrika* 82, 561–574.

Hansen, Peter R., and Asger Lunde, 2005, A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH (1, 1)?, *Journal of Applied Econometrics* 20, 873–889.

Hendry, David F, 1995, *Dynamic Econometrics* (Oxford: Oxford University Press).

Hendry, David F, and Hans-Martin Krolzig, 2001, *Automatic econometric model selection using PcGets* (London: Timberlake Consultants).

Hill, Bruce M, 1975, A simple general approach to Inference about the Tail of a Distribution, *Annals of Statistics* 1163–1174.

Hollstein, Fabian, and Marcel Prokopczuk, 2016, Estimating Beta, *Journal of Financial and Quantitative Analysis* 51, 1437–1466.

Hollstein, Fabian, Marcel Prokopczuk, and Chardin Wese Simen, 2019, Estimating Beta: Forecast Adjustments and the Impact of Stock Characteristics for a Broad Cross-Section, *Journal of Financial Markets* 44, 91–118.

Jackwerth, Jens Carsten, 2000, Recovering Risk Aversion from Option Prices and Realized Returns, *Review of Financial Studies* 13, 433–451.

Jiang, George J., and Yisong S. Tian, 2005, The Model-Free Implied Volatility and its Information Content, *Review of Financial Studies* 18, 1305–1342.

Kelly, Bryan, and Hao Jiang, 2014, Tail Risk and Asset Prices, *Review of Financial Studies* 27, 2841–2871.

Lahiri, Soumendra N., 1999, Theoretical Comparisons of Block Bootstrap Methods, *Annals of Statistics* 386–404.

Lin, Huidi, and Viktor Todorov, 2019, Aggregate Asymmetry in Idiosyncratic Jump Risk, *Working Paper* .

Lindeman, Richard H., PF. Merenda, and Ruth Z. Gold, 1980, Introduction to Bivariate and Multivariate Analysis, *New York: Scott, Foresman and Co* .

Lu, Zhongjin, and Scott Murray, 2019, Bear Beta, *Journal of Financial Economics* 131, 736–760.

Maheu, John M., Thomas H. McCurdy, and Xiaofei Zhao, 2013, Do Jumps Contribute to the Dynamics of the Equity Premium?, *Journal of Financial Economics* 110, 457–477.

Mancini, Cecilia, 2001, Disentangling the Jumps of the Diffusion in a geometric jumping Brownian Motion, *Giornale dell'Istituto Italiano degli Attuari* 64, 44.

Martin, Ian, 2017, What is the Expected Return on the Market?, *Quarterly Journal of Economics* 132, 367–433.

Muir, Tyler, 2017, Financial crises and risk premia, *Quarterly Journal of Economics* 132, 765–809.

Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 53, 1047–1070.

Nicholls, Desmond F., and Alun L. Pope, 1988, Bias in the Estimation of Multivariate Autoregressions, *Australian Journal of Statistics* 30, 296–309.

Rapach, David E., Jack K. Strauss, and Guofu Zhou, 2013, International Stock Return Predictability: What is the Role of the United States?, *Journal of Finance* 68, 1633–1662.

Rietz, Thomas A., 1988, The equity risk premium a solution, *Journal of Monetary Economics* 22, 117 – 131.

Seo, Sang Byung, and Jessica A. Wachter, 2018, Option Prices in a Model with Stochastic Disaster Risk, *Management Science* 65, 3470–3469.

Stambaugh, Robert F., 1999, Predictive Regressions, *Journal of Financial Economics* 54, 375–421.

Symitsi, Efthymia, Lazaros Symeonidis, Apostolos Kourtis, and Raphael Markellos, 2018, Covariance forecasting in equity markets, *Journal of Banking & Finance* 96, 153–168.

Vilkov, Grigory, and Yan Xiao, 2015, Option-implied Information and Predictability of Extreme Returns, *SAFE Working Paper* .

Weller, Brian, 2018, Measuring Tail Risks at High Frequency, *Review of Financial Studies* 32, 3571–3616.

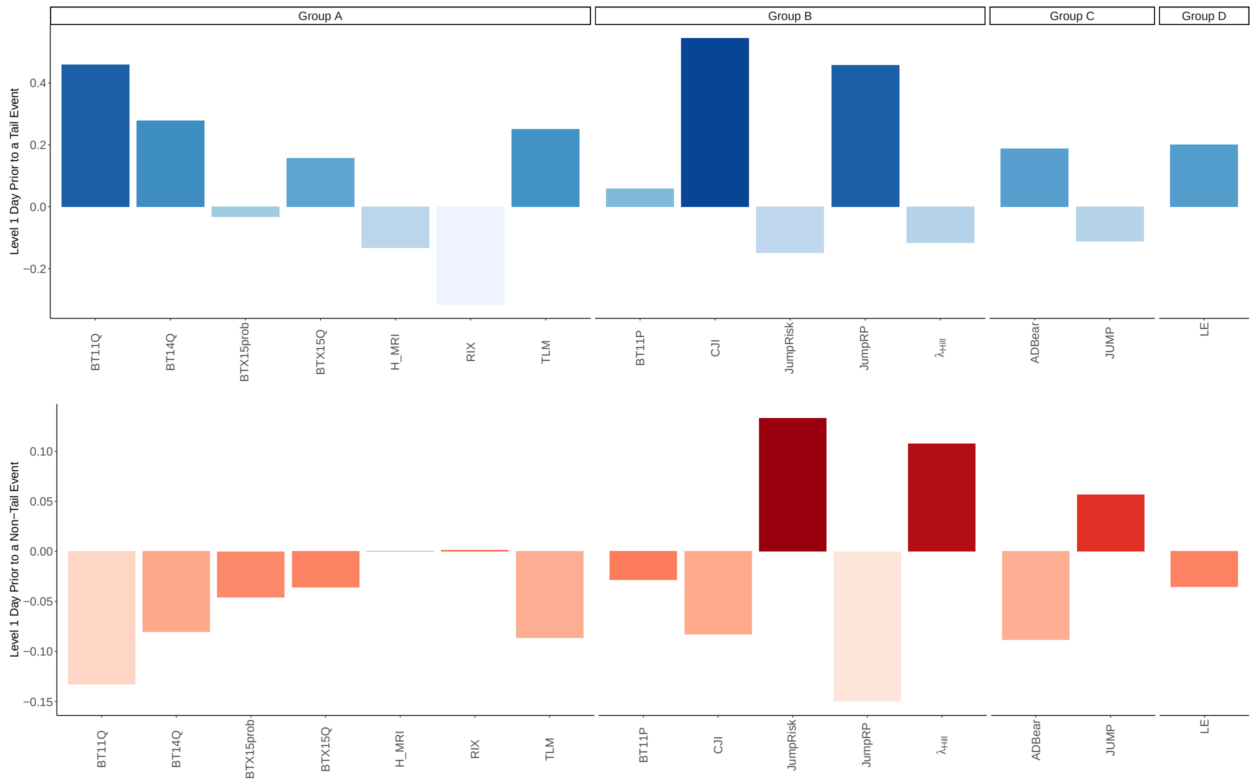

Figure 1. The upper panel of this figure displays the average levels of different tail risk measures one day before (two-sigma or more) left tail events. In the lower panel, we display a simple placebo test that shows the average level of the tail risk measures ahead of (absolute return of 0.02 sigma or less) non-tail events. All tail risk measures are standardized to have a mean of zero and a volatility of one. We separate the tail risk measures into four groups: Option-Implied (Group A), Stock-Return-Based (Group B), Option-Return Based (Group C), and Macroeconomic Measures (Group D). The colors indicate the intensity of the tail risk measures ahead of the events. The definitions of the tail risk measure acronyms are in Table I.

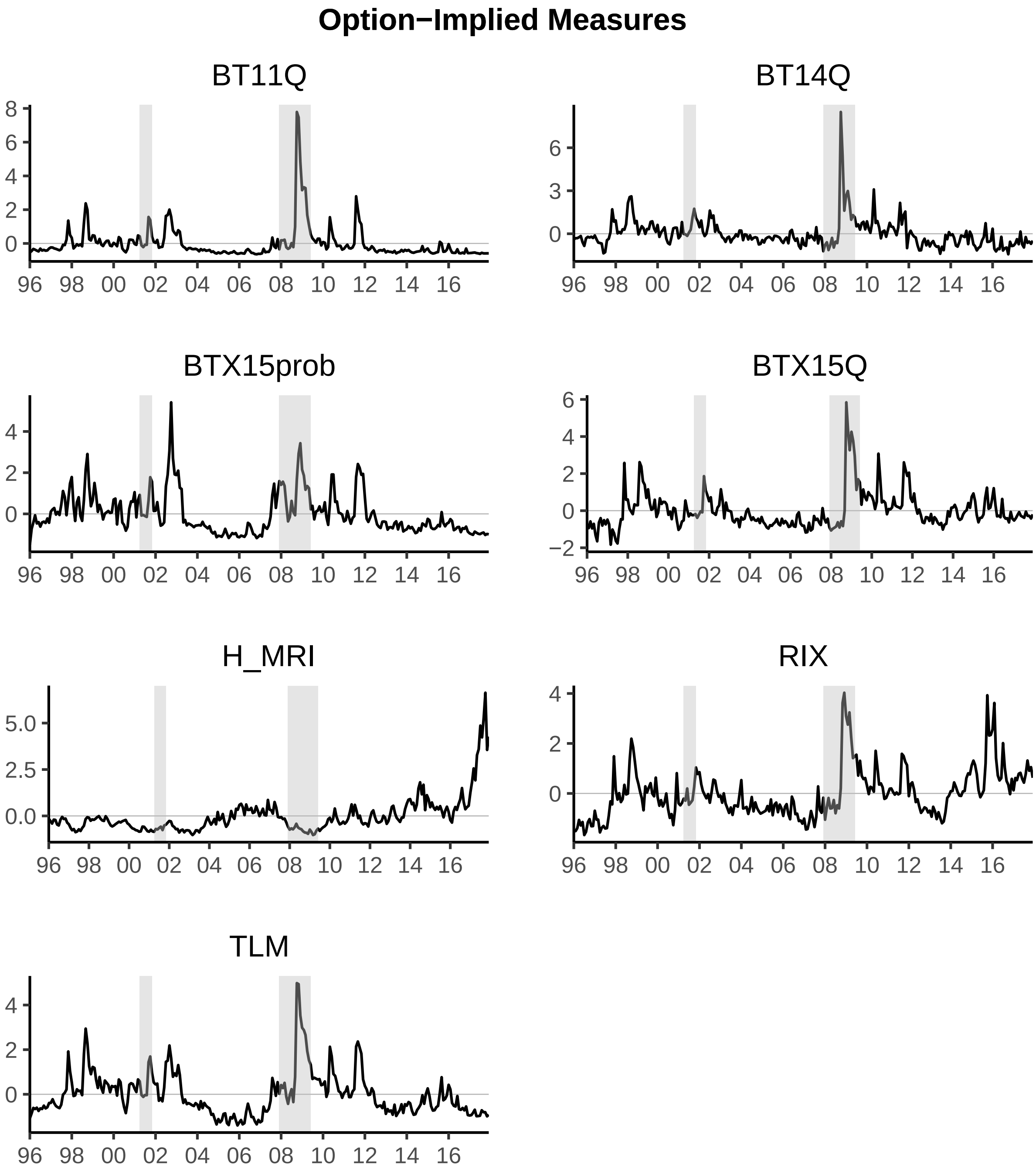

Figure 2. This figure displays the time-series of the standardized (mean of zero and standard deviation of one) option-implied tail risk measures. For a better visualization, we average all daily observations of the tail risk measures during a month. The shaded areas indicate business cycle contractions as identified by the NBER. The definitions of the tail risk measure acronyms are given in Table I.

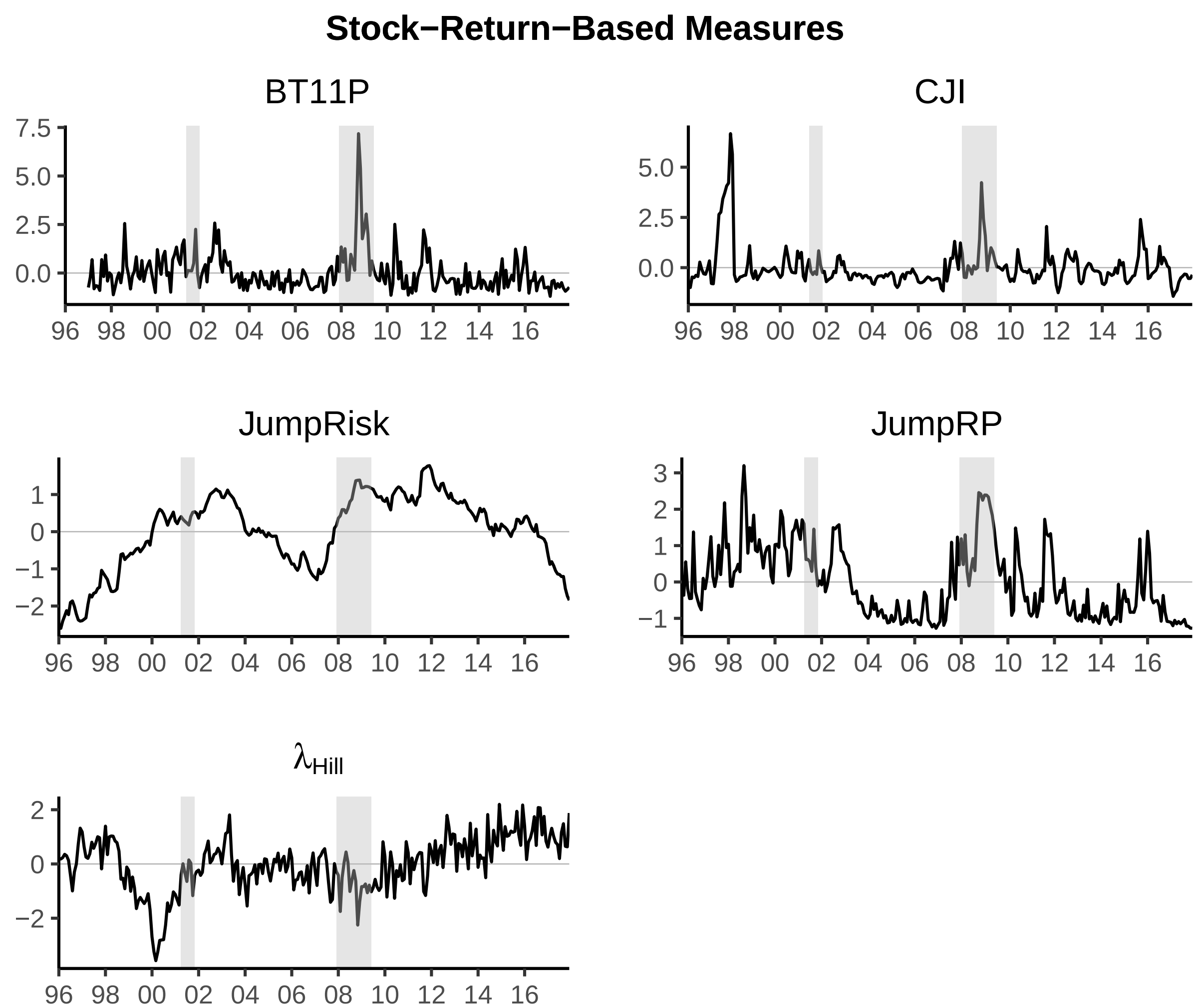

Figure 3. This figure displays the time-series of the standardized (mean of zero and standard deviation of one) stock-return-based tail risk measures. For a better visualization, we average all daily observations of the tail risk measures during a month. The shaded areas indicate business cycle contractions as identified by the NBER. The definitions of the tail risk measure acronyms are given in Table I.

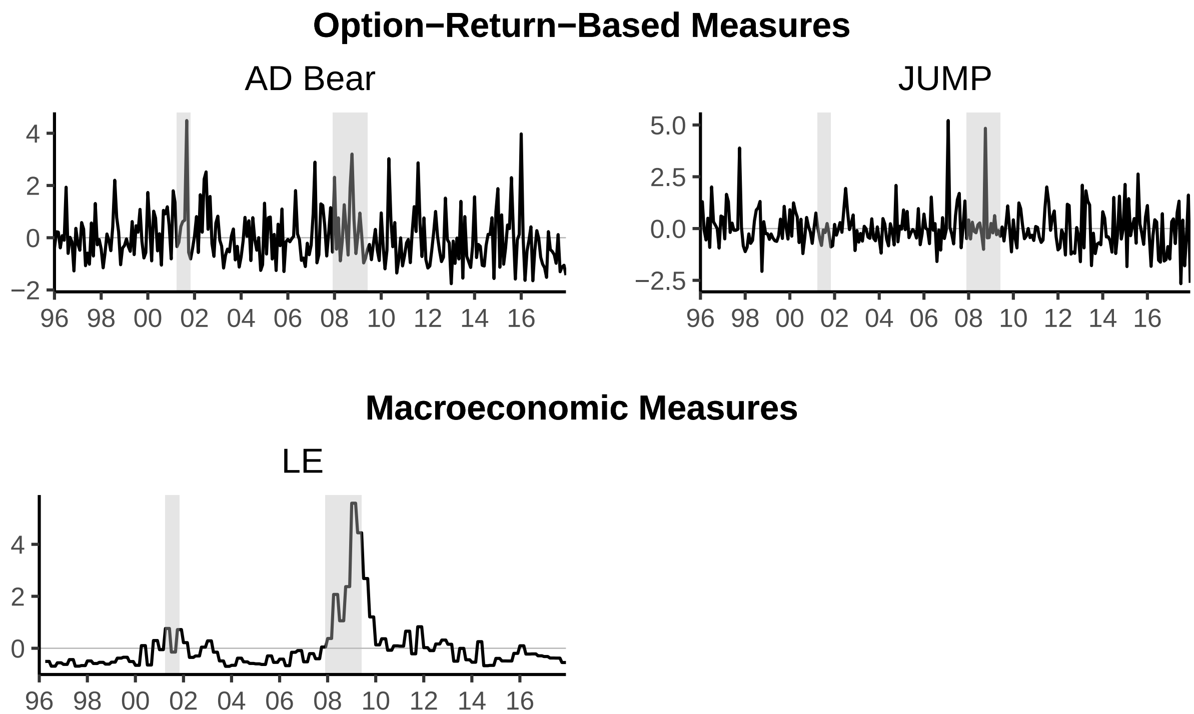

Figure 4. This figure displays the time-series of the standardized (mean of zero and standard deviation of one) option-return-based- and macroeconomic tail risk measures. For a better visualization, we average all daily observations of the tail risk measures during a month. The shaded areas indicate business cycle contractions as identified by the NBER. The definitions of the tail risk measure acronyms are given in Table I.

Table I Description of Tail Risk Measures

This table presents the main tail risk measures used in this study. The column "Source" provides the original paper, "Description" delivers a brief sketch of the approach. "Acronym" defines the symbol used in this paper to refer to the measure. "Interpretation" characterizes the main concepts underlying the measure. We allocate the measures to the different categories to which they most naturally (but rather broadly defined) belong. "Estimation" characterizes how the parameters are estimated: "OS" and "IS" denote the characterization as an out-of-sample (available in real-time) or an in-sample (parameters optimized using forward-looking data) measure, respectively. Finally, "Freq" denotes the frequency at which the different measures are available. "D" indicates that we observe a measure every trading day. "W", "M" and "Q" denote weekly, monthly and quarterly observation frequencies, respectively.

Source	Description	Acronym	Interpretation	Estimation	Freq
Group A - Option-Implied Measures					
Bollerslev and Todorov (2011b)	Left tail approximation measure under $\mathbb{Q}$	BT11Q	Expected Shortfall	OS	D
Bollerslev and Todorov (2014)	Weekly parametric estimate of time-varying left tail measure	BT14Q	Expected Shortfall	OS	W
Bollerslev et al. (2015)	Probability of a daily loss of 10% or more	BTX15prob	Tail Probability	OS	D
Bollerslev et al. (2015)	Daily non-parametric estimate of time-varying left tail measure	BTX15Q	Expected Shortfall	OS	D
Gormsen and Jensen (2020)	First PC of risk-neutral higher moments	H_MRI	Higher Moment Risk	OS	D
Gao et al. (2018) and Gao et al. (2019)	Left tail volatility as the difference in two volatility indexes	RIX	Left Tail Volatility	OS	M
Vilkov and Xiao (2015)	Expected shortfall inferred from parametrised tail distribution	TLM	Expected Shortfall	OS	D
Group B - Stock-Return-Based Measures					
Bollerslev and Todorov (2011b)	Left-tail approximation measure under $\mathbb{P}$	BT11P	Expected Shortfall	OS	D
Christoffersen et al. (2012)	Parametric model-implied jump intensity	CJI	Jump Intensity	OS	D
Maheu et al. (2013)	Parametric model-implied jump intensity	JumpRisk	Jump Intensity	OS	D
Maheu et al. (2013)	Parametric model-implied jump risk premium	JumpRP	Jump Risk Premium	OS	D
Kelly and Jiang (2014)	Left tail of the cross-section of stock returns	λ_{Hill}	Expected Shortfall	OS	M
Group C - Option-Return-Based Measures					
Lu and Murray (2019)	Return of bear spread put option positions	ADBear	Jump Risk Premium	OS	D
Cremers et al. (2015)	Return of vega-neutral, gamma-positive option portfolio	JUMP	Jump Risk Premium	OS	D
Group D - Macroeconomic Measures					
Adrian et al. (2019)	Left entropy of expected GDP growth	LE	Left Entropy	OS	Q

Table II Summary Statistics

This table displays the summary statistics of the tail risk measures considered. The definitions of the tail risk measure acronyms are given in Table I. We sort the tail risk measures into different categories based on their underlying data. We present several time-series statistics. "*Mean*" denotes the time-series average, "*SD*" is the standard deviation. For the remainder of the paper, we standardize the tail risk measures to have a mean of zero and a standard deviation of one. "*Median*", "*Min*" and "*Max*" denote the median, the lowest and the highest values, respectively, attained by the measures. "*Skewness*" and "*Kurtosis*" denote the skewness and kurtosis of the measures' distributions. Finally, "$AR(1)$" denotes the first-order autocorrelation of the measures. All measures except for RIX, $BT14Q$, λ_{Hill} and LE are available at the daily frequency. $BT14Q$ is weekly, λ_{Hill} and RIX are monthly, and LE is quarterly. $BT11P$ is scaled by 100.

	Mean	*SD*	*Median*	*Min*	*Max*	*Skewness*	*Kurtosis*	$AR(1)$
Group A - Option-Implied Measures								
$BT11Q$	0.3962	0.6099	0.2144	0.0060	10.4551	5.2393	45.0622	0.9280
$BT14Q$	0.0086	0.0046	0.0076	0.0023	0.0579	3.4557	28.2029	0.6107
$BTX15prob$	0.8299	0.5640	0.6490	0.0000	4.5502	1.5709	6.1317	0.9973
$BTX15Q$	0.0789	0.0359	0.0703	0.0021	0.3985	2.4158	13.4343	0.9320
H_MRI	0.0000	1.9152	−0.4398	−2.3253	17.5014	3.5753	21.2163	0.9730
RIX	0.1572	0.0205	0.1545	0.1230	0.2402	1.3616	5.7250	0.8154
TLM	0.0437	0.0146	0.0403	0.0218	0.1628	1.8851	9.4089	0.9770
Group B - Stock-Return-Based Measures								
$BT11P$	0.0058	0.0072	0.0038	0.0000	0.0864	2.6368	15.7063	0.0490
CJI	0.0152	0.0228	0.0095	−0.0194	0.1727	2.9916	15.0090	0.9823
$JumpRisk$	0.1596	0.0272	0.1651	0.0885	0.2089	−0.5478	2.4405	0.9988
$JumpRP$	0.7400	0.3102	0.6585	0.3556	1.9788	0.8626	2.9707	0.9646
λ_{Hill}	0.4426	0.0275	0.4450	0.3447	0.5054	−0.5789	3.8619	0.7538
Group C - Option-Return-Based Measures								
$ADBear$	−0.0963	0.7638	−0.3089	−0.9950	10.1970	2.8190	18.9207	0.6775
$JUMP$	−0.0019	0.0518	−0.0083	−0.8375	1.2189	4.6072	99.0955	−0.0401
Group D - Macroeconomic Measures								
LE	0.0885	0.1690	0.0331	−0.0266	1.0478	3.5552	17.7338	0.7904

Table III Correlations

This table displays the time-series correlations among the tail risk measures considered. The definitions of the tail risk measure acronyms are given in Table I. In order to compare the correlations, we use a daily sample with constant extrapolation. The last line shows the correlation with the VIX.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Group A - Option-Implied Measures															
(1) $BT11Q$		0.66	0.64	0.70	−0.28	0.39	0.87	0.37	0.38	0.33	0.61	−0.26	0.25	0.09	0.49
(2) $BT14Q$			0.42	0.62	−0.12	0.30	0.67	0.17	0.26	0.19	0.47	−0.22	0.14	0.03	0.33
(3) $BTX15prob$				0.44	−0.44	0.33	0.79	0.16	0.28	0.37	0.63	−0.28	0.05	0.02	0.36
(4) $BTX15Q$					−0.05	0.56	0.77	0.22	0.25	0.33	0.48	−0.12	0.14	0.05	0.45
(5) H_MRI						0.14	−0.36	−0.13	−0.07	−0.41	−0.30	0.40	−0.16	−0.11	−0.25
(6) RIX							0.49	0.07	0.09	0.32	0.29	0.04	−0.04	−0.03	0.44
(7) TLM								0.32	0.37	0.43	0.74	−0.30	0.24	0.09	0.51
Group B - Stock-Return-Based Measures															
(8) $BT11P$									0.08	0.12	0.34	−0.09	0.44	0.33	0.13
(9) CJI										−0.02	0.47	0.04	0.08	0.01	0.09
(10) $JumpRisk$											0.19	−0.22	0.07	0.03	0.47
(11) $JumpRP$												−0.35	0.32	0.12	0.36
(12) λ_{Hill}													−0.05	−0.03	−0.22
Group C - Option-Return-Based Measures															
(13) $ADBear$														0.23	0.01
(14) $JUMP$															0.01
Group D - Macroeconomic Measures															
(15) LE															
ρ_{VIX}	0.89	0.64	0.80	0.69	−0.49	0.41	0.96	0.36	0.44	0.38	0.85	−0.35	0.26	0.08	0.53

Table IV Principal Components

This table presents the results of a principal component analysis (PCA) of the standardized tail risk measures. The definitions of the tail risk measure acronyms are given in Table I. We use a daily sample of the tail risk measures, with constant extrapolation. We display the first two PCs among all measures and within the different subgroups. The column "$CumVar$" displays the cumulative variance explained by the PCs. The last column displays the correlation of each PC with the VIX.

	$BT11Q$	$BT14Q$	$BTX15prob$	$BTX15Q$	H_MRI	RIX	TLM	$BT11P$	CJI	$JumpRisk$	$JumpRP$	λ_{Hill}	$ADBear$	$JUMP$	LE	$CumVar$	ρ_{VIX}
Full Sample																	
PC1	0.37	0.30	0.32	0.32	−0.19	0.21	0.40	0.16	0.18	0.20	0.35	−0.15	0.12	0.05	0.25	0.38	0.96
PC2	−0.01	−0.08	−0.06	−0.18	−0.25	−0.40	−0.04	0.44	0.10	−0.14	0.17	−0.12	0.49	0.41	−0.25	0.49	0.05
Group A - Option-Implied Measures																	
PC1	0.44	0.38	0.38	0.41	−0.17	0.29	0.48									0.58	0.94
PC2	−0.03	0.05	−0.27	0.26	0.76	0.52	−0.06									0.76	-0.23
Group B - Stock-Return-Based Measures																	
PC1								0.41	0.38	0.31	0.66	−0.40				0.37	0.84
PC2								−0.03	−0.69	0.56	−0.12	−0.45				0.59	-0.04
Group C - Option-Return-Based Measures																	
PC1													0.71	0.71		0.61	0.21
PC2													−0.71	0.71		1.00	-0.14

Table V Prediction of Tail Events

This table presents the coefficients from the predictive probit regressions. We perform single probit regressions of a dummy variable on each lagged tail risk measure:

$$D_{t+\Delta t} = a + b \cdot TRM_t + \epsilon_{t+\Delta t}.$$

$D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. The conditional volatility is defined as the level of the VIX at the end of the previous day. TRM_t is the current observation of a tail risk measure. We use three different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), and (iii) one-month (*Monthly*). In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. The columns R^2 present the McFadden R^2s, multiplied by 100. "*PCOneAll*", "*PCOneOption*", "*PCOneStReturn*", and "*PCOneOpReturn*" denote the first PCs of all measures, option-implied, stock-return-based, and option-return-based tail risk measures, respectively. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2
Group A - Option-Implied Measures						
BT11Q	0.09^{***}	1.22	0.08	0.68	0.03	0.08
	(0.021)		(0.048)		(0.046)	
BT14Q	0.07	0.53	0.08	0.56	0.08^{**}	0.68
	(0.042)		(0.048)		(0.041)	
BTX15prob	0.00	0.01	−0.04	0.08	-0.12^{***}	0.60
	(0.060)		(0.079)		(0.040)	
BTX15Q	0.04	0.17	0.04	0.12	0.00	0.01
	(0.072)		(0.073)		(0.055)	
H_MRI	−0.24	1.46	−0.12	0.49	−0.28	1.76
	(0.183)		(0.092)		(0.271)	
RIX	−0.14	1.08	−0.09	0.55	-0.16^{**}	1.13
	(0.132)		(0.106)		(0.076)	
TLM	0.08	0.55	0.06	0.28	0.04	0.14
	(0.058)		(0.084)		(0.054)	
Group B - Stock-Return-Based Measures						
BT11P	0.02	11.37	0.02	0.83	0.05	1.05
	(0.072)		(0.055)		(0.049)	
CJI	0.12^{***}	1.75	0.09	0.81	0.08	0.49
	(0.044)		(0.088)		(0.074)	
JumpRisk	−0.03	0.07	0.02	0.03	0.35^{**}	4.82
	(0.107)		(0.102)		(0.144)	
JumpRP	0.16^{**}	2.06	0.05	0.20	0.11	0.92
	(0.070)		(0.100)		(0.121)	
λ_{Hill}	−0.05	0.24	−0.04	0.18	0.08	0.39
	(0.070)		(0.081)		(0.073)	
Group C - Option-Return-Based Measures						
ADBear	0.06	0.32	0.11^{**}	1.15	0.05	0.22
	(0.049)		(0.053)		(0.061)	
JUMP	−0.04	0.11	−0.03	0.06	0.05^{**}	0.27
	(0.055)		(0.040)		(0.027)	
Group D - Macroeconomic Measures						
LE	0.06	4.05	−0.01	0.20	0.07	0.63
	(0.055)		(0.092)		(0.069)	
PCOneAll	0.12^{**}	12.57	0.05	1.01	0.06	1.09
	(0.047)		(0.092)		(0.063)	
PCOneOption	0.06	0.43	0.05	0.27	0.02	0.08
	(0.057)		(0.076)		(0.044)	
PCOneStReturn	0.15^{**}	13.17	0.06	1.12	0.12	1.87
	(0.064)		(0.110)		(0.111)	
PCOneOpReturn	0.02	0.05	0.06	0.36	0.06	0.38
	(0.046)		(0.042)		(0.043)	

Table VI Multiple Prediction of Tail Events

This table presents the coefficients from the predictive probit regressions. We perform multiple probit regressions of a dummy variable on lagged tail risk measures:

$$D_{t+\Delta t} = a + b \cdot TRM_t + \epsilon_{t+\Delta t}.$$

$D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. The conditional volatility is defined as the level of the VIX at the end of the previous day. TRM_t is a vector of the current observations of the tail risk measures. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), and (iii) one-month (*Monthly*). For each forecast horizon, we first perform variable selection based on the PcGets algorithm. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. The columns R^2 present the partial McFadden R^2s, obtained by dominance analysis, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2
Group A - Option-Implied Measures						
$BT11Q$	0.06^{**}	1.09				
	(0.027)					
$BT14Q$			0.12^{**}	0.78	0.02	0.68
			(0.054)		(0.121)	
$BTX15prob$			-0.07	0.19	-0.45^{***}	1.77
			(0.088)		(0.144)	
$BTX15Q$						
H_MRI						
RIX			-0.14	0.90	-0.27^{*}	1.99
			(0.140)		(0.138)	
TLM					0.35	1.07
					(0.217)	
Group B - Stock-Return-Based Measures						
$BT11P$						
CJI	0.11^{**}	1.72				
	(0.047)					
JumpRisk						
JumpRP						
λ_{Hill}						
Group C - Option-Return-Based Measures						
$ADBear$						
$JUMP$						
Group D - Macroeconomic Measures						
LE						
Controls	*Yes*		*Yes*		*Yes*	

Table VII Predictability of Left Tail Variation

This table presents the coefficients from a predictive regression for future left tail variation. We perform single regressions of the standardized realized left tail variation on each lagged tail risk measure:

$$LTV^{\mathbb{P}}_{t+\Delta t} = a + b \cdot TRM_t + c \cdot LTV^{\mathbb{P}}_t + d \cdot VIX_t + \epsilon_{t+\Delta t}.$$

TRM_t is the current observation of a tail risk measure. We control for the lagged left tail variation $LTV^{\mathbb{P}}_t$ and the current level of the VIX (VIX_t). We use three different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), and (iii) one-month (*Monthly*). In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. "*PCOneAll*", "*PCOneOption*", "*PCOneStReturn*", and "*PCOneOpReturn*" denote the first PCs of all measures, option-implied, stock-return-based, and option-return-based tail risk measures, respectively. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2
Group A - Option-Implied Measures						
BT11Q	0.19^{**}	2.88	0.30^{**}	8.52	0.14	9.27
	(0.098)		(0.173)		(0.140)	
BT14Q	0.10^{*}	2.18	0.18^{*}	6.67	0.09	6.05
	(0.070)		(0.133)		(0.083)	
BTX15prob	-0.12^{**}	0.96	-0.23^{*}	2.99	-0.23^{**}	3.40
	(0.057)		(0.152)		(0.183)	
BTX15Q	0.09^{**}	1.95	0.13	5.42	0.02	4.77
	(0.053)		(0.111)		(0.093)	
H_MRI	0.03^{*}	0.31	0.05	0.97	−0.01	1.35
	(0.020)		(0.050)		(0.041)	
RIX	−0.03	0.19	−0.06	0.60	−0.06	0.86
	(0.028)		(0.062)		(0.081)	
TLM	0.05	1.99	0.14	6.39	−0.07	6.73
	(0.088)		(0.219)		(0.271)	
Group B - Stock-Return-Based Measures						
BT11P	−0.01	0.23	0.04	1.30	0.06^{**}	1.91
	(0.046)		(0.029)		(0.034)	
CJI	0.02	0.62	0.04	1.84	0.03	2.30
	(0.030)		(0.053)		(0.055)	
JumpRisk	0.03^{**}	0.63	0.06^{**}	1.95	0.09^{**}	3.50
	(0.014)		(0.030)		(0.076)	
JumpRP	-0.09^{*}	1.18	−0.15	3.64	−0.03	4.18
	(0.053)		(0.164)		(0.134)	
λ_{Hill}	0.01	0.21	0.01	0.68	0.00	1.03
	(0.018)		(0.037)		(0.050)	
Group C - Option-Return-Based Measures						
ADBear	0.01	0.22	0.02	0.64	0.06^{**}	1.02
	(0.026)		(0.038)		(0.049)	
JUMP	0.04^{*}	0.24	0.02	0.10	0.03^{***}	0.16
	(0.031)		(0.014)		(0.021)	
Group D - Macroeconomic Measures						
LE	0.03	0.89	0.06^{*}	2.81	0.14^{***}	5.83
	(0.024)		(0.041)		(0.063)	
PCOneAll	0.20^{**}	2.30	0.32^{**}	7.12	0.29^{**}	8.47
	(0.099)		(0.145)		(0.191)	
PCOneOption	0.14	2.27	0.19	6.78	−0.05	7.08
	(0.114)		(0.223)		(0.301)	
PCOneStReturn	−0.02	1.37	0.02	4.66	0.13	6.47
	(0.064)		(0.111)		(0.119)	
PCOneOpReturn	0.04	0.35	0.02	0.51	0.06^{**}	0.81
	(0.031)		(0.032)		(0.040)	
Controls	*Yes*		*Yes*		*Yes*	

Table VIII Multiple Predictability of Left Tail Variation

This table presents the coefficients from a predictive regression for future left tail variation. We perform multiple regressions of the realized left tail variation on the lagged tail risk measures:

$$LTV^{\mathbb{P}}_{t+\Delta t} = a + b \cdot TRM_t + c \cdot LTV^{\mathbb{P}}_t + d \cdot VIX_t + \epsilon_{t+\Delta t}.$$

TRM_t is a vector of the current observations of the tail risk measures. We control for the lagged left tail variation $LTV^{\mathbb{P}}_t$ and the current level of the VIX (VIX_t). We use three different forecast horizons Δt: (i) one-day ($Daily$), (ii) one-week ($Weekly$), and (iii) one-month ($Monthly$). For each forecast horizon, we first perform variable selection based on the PcGets algorithm. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2
Group A - Option-Implied Measures						
$BT11Q$	0.19^{***}	4.15			-0.02^{**}	10.27
	(0.046)				(0.008)	
$BT14Q$						
$BTX15prob$						
$BTX15Q$						
H_MRI						
RIX						
TLM						
Group B - Stock-Return-Based Measures						
$BT11P$						
CJI						
JumpRisk	0.04^{***}	0.65				
	(0.015)					
JumpRP						
λ_{Hill}						
Group C - Option-Return-Based Measures						
$ADBear$						
$JUMP$						
Group D - Macroeconomic Measures						
LE			0.04^{***}	3.12	0.01^{***}	4.76
			(0.010)		(0.004)	
Controls	*Yes*		*Yes*		*Yes*	

Table IX Return Predictability

This table presents the coefficients from a return predictability regression. We perform single regressions of the market excess returns on each lagged tail risk measure:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot Controls_t + \epsilon_{t+\Delta t}.$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is the current observation of a tail risk measure. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), (iii) one-month (*Monthly*), and (iv) one-year (*Annually*). In parentheses, we present robust Newey and West (1987) standard errors with lag length chosen to be the maximum of 29 and the number of overlapping observations. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. "*PCOneAll*", "*PCOneOption*", "*PCOneStReturn*", and "*PCOneOpReturn*" denote the first PCs of all measures, option-implied, stock-return-based, and option-return-based tail risk measures, respectively. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2	*Annually*	R^2
Group A - Option-Implied Measures								
BT11Q	35.96^{***}	0.52	15.21^{**}	0.51	6.64^{*}	0.48	2.85^{*}	2.17
	(7.350)		(5.613)		(4.504)		(1.915)	
BT14Q	5.15	0.02	0.70	0.02	-4.18^{*}	0.34	0.71	0.56
	(6.213)		(4.386)		(2.742)		(1.713)	
BTX15prob	11.67^{**}	0.07	7.40	0.24	8.41^{**}	1.18	−2.13	0.49
	(6.121)		(5.518)		(4.242)		(3.193)	
BTX15Q	1.50	0.01	−3.19	0.03	−3.37	0.14	1.14	1.61
	(5.816)		(5.227)		(3.141)		(2.337)	
H_MRI	-7.23^{**}	0.02	−2.98	0.04	−0.65	0.09	3.10	1.01
	(4.028)		(3.313)		(2.917)		(2.620)	
RIX	2.48	0.02	3.16	0.13	3.35	0.61	0.25	1.32
	(5.927)		(5.402)		(4.528)		(3.315)	
TLM	26.51^{***}	0.26	13.48^{**}	0.50	4.11	0.50	−0.73	0.77
	(8.577)		(6.277)		(4.563)		(2.825)	
Group B - Stock-Return-Based Measures								
BT11P	25.95^{***}	0.63	13.89^{***}	0.96	3.91^{***}	0.29	0.58	0.10
	(5.859)		(2.855)		(1.383)		(0.394)	
CJI	3.75	0.01	1.49	0.03	−0.03	0.05	1.22	0.70
	(4.335)		(4.330)		(2.941)		(1.285)	
JumpRisk	−5.67	0.01	-7.73^{*}	0.05	-10.91^{**}	0.55	-15.36^{***}	12.30
	(5.754)		(5.798)		(5.197)		(2.654)	
JumpRP	15.94^{***}	0.11	11.29^{**}	0.38	4.72	0.52	−2.14	0.46
	(5.360)		(4.721)		(3.997)		(2.319)	
λ_{Hill}	−0.42	0.00	1.36	0.05	0.04	0.08	6.25^{***}	9.38
	(4.057)		(4.015)		(3.469)		(1.610)	
Group C - Option-Return-Based Measures								
ADBear	19.16^{***}	0.39	13.31^{***}	1.07	3.30^{**}	0.30	−0.10	0.01
	(4.820)		(3.073)		(1.565)		(0.381)	
JUMP	2.83	0.01	5.80^{***}	0.20	1.14^{*}	0.03	0.20	0.01
	(6.084)		(1.585)		(0.703)		(0.141)	
Group D - Macroeconomic Measures								
LE	2.20	0.01	0.45	0.03	3.14	0.09	−2.27	0.49
	(7.517)		(7.774)		(5.657)		(2.816)	
PCOneAll	34.45^{***}	0.29	18.66^{***}	0.54	8.19^{*}	0.68	1.18	1.38
	(8.736)		(6.265)		(4.955)		(2.975)	
PCOneOption	22.46^{***}	0.18	9.16^{*}	0.26	2.74	0.33	0.52	1.21
	(8.518)		(6.161)		(4.788)		(3.112)	
PCOneStReturn	24.42^{***}	0.25	14.23^{***}	0.49	5.39^{*}	0.41	−2.27	0.79
	(5.310)		(4.605)		(3.644)		(1.983)	
PCOneOpReturn	14.06^{***}	0.21	12.22^{***}	0.89	2.84^{**}	0.22	0.06	0.00
	(5.477)		(2.523)		(1.294)		(0.271)	
Controls	*Yes*		*Yes*		*Yes*		*Yes*	

Table X Multiple Return Predictability

This table presents the coefficients from a return predictability regression. We perform multiple regressions of the market excess returns on lagged tail risk measures:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot Controls_t + \epsilon_{t+\Delta t}.$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is a vector of the current observations of the tail risk measures. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), (iii) one-month (*Monthly*), and (iv) one-year (*Annually*). For each forecast horizon, we first perform variable selection based on the PcGets selection algorithm. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with lag length chosen to be the maximum of 29 and the number of overlapping observations. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2	*Annually*	R^2
Group A - Option-Implied Measures								
BT11Q	47.61***	0.53			10.47*	0.57	6.35***	6.42
	(10.620)				(5.865)		(0.831)	
BT14Q					−6.45**	0.63		
					(2.499)			
BTX15prob					8.32**	1.31		
					(4.793)			
BTX15Q	−21.81***	0.12	−21.65***	0.44	−8.22***	0.36		
	(8.673)		(6.785)		(3.136)			
H_MRI								
RIX								
TLM			22.23***	0.53				
			(7.645)					
Group B - Stock-Return-Based Measures								
BT11P	17.39***	0.46	7.35***	0.51				
	(5.906)		(2.697)					
CJI								
JumpRisk							−13.26***	7.93
							(3.057)	
JumpRP								
λ_{Hill}								
Group C - Option-Return-Based Measures								
ADBear			8.78***	0.70				
			(3.063)					
JUMP								
Group D - Macroeconomic Measures								
LE								
Controls	*Yes*		*Yes*		*Yes*		*Yes*	

Table XI Tail Return Predictability

This table presents the coefficients from a return predictability regression. We perform single regressions of the market excess returns on each lagged tail risk measure, while separating crash and non-crash periods:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot TRM_t \cdot D_{t+\Delta t} + d \cdot D_{t+\Delta t} + e \cdot Controls_t + \epsilon_{t+\Delta t},$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is the current observation of a tail risk measure. $D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use three different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), and (iii) one-month (*Monthly*). In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. "*PCOneAll*", "*PCOneOption*", "*PCOneStReturn*", and "*PCOneOpReturn*" denote the first PCs of all measures, option-implied, stock-return-based, and option-return-based tail risk measures, respectively. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily				Weekly				Monthly			
	b	c	$R^2(b)$	$R^2(c)$	b	c	$R^2(b)$	$R^2(c)$	b	c	$R^2(b)$	$R^2(c)$
Group A - Option-Implied Measures												
$BT11Q$	40.39***	−338.62***	0.64	2.25	17.20***	−136.04***	0.63	2.29	6.04	−77.01***	0.44	1.11
	(7.059)	(45.081)			(5.126)	(13.557)			(4.437)	(10.087)		
$BT14Q$	8.79*	−277.34***	0.05	1.37	2.01	−109.26***	0.03	1.30	-3.57	−20.61**	0.27	0.93
	(5.773)	(46.027)			(3.863)	(39.328)			(2.680)	(10.689)		
$BTX15prob$	9.52*	−487.18***	0.07	0.91	6.61	−166.08***	0.23	0.38	6.80*	−153.63***	0.98	2.29
	(5.791)	(90.487)			(5.175)	(46.441)			(3.909)	(49.487)		
$BTX15Q$	4.46	−324.63***	0.02	1.34	-2.62	−139.01***	0.03	1.14	-3.52	-19.10	0.15	0.05
	(5.797)	(31.139)			(4.776)	(43.157)			(2.949)	(18.197)		
H_MRI	−10.09**	460.77***	0.03	1.74	-4.22	223.13**	0.06	1.86	-1.61	83.69***	0.10	3.36
	(4.396)	(143.909)			(3.209)	(93.710)			(2.699)	(17.492)		
RIX	-1.09	−251.02***	0.01	0.20	0.88	-53.51	0.11	0.14	2.01	76.79*	0.48	3.42
	(5.435)	(39.112)			(4.735)	(60.278)			(3.811)	(48.160)		
TLM	29.90***	−380.12***	0.32	1.95	14.76***	−148.76***	0.57	1.69	4.05	−76.75***	0.49	1.31
	(8.310)	(29.557)			(5.744)	(26.982)			(4.352)	(5.684)		
Group B - Stock-Return-Based Measures												
$BT11P$	26.24***	−152.49*	0.64	0.14	14.32***	−74.21**	1.01	0.32	4.16***	−14.01***	0.33	0.27
	(5.347)	(86.166)			(2.740)	(32.716)			(1.296)	(2.929)		
CJI	8.37*	−313.94***	0.05	2.15	3.69	−93.70***	0.07	1.68	0.68	−47.02***	0.06	1.47
	(3.834)	(40.887)			(3.338)	(14.757)			(2.700)	(5.383)		
$JumpRisk$	-2.24	−206.95**	0.00	0.25	-5.31	−122.78***	0.03	0.77	−8.43**	−51.40**	0.36	4.28
	(5.583)	(69.684)			(5.252)	(44.655)			(4.594)	(26.137)		
$JumpRP$	20.69***	−374.10***	0.17	2.36	12.05***	−150.32***	0.43	1.57	6.01	−54.31***	0.62	2.19
	(5.216)	(51.523)			(4.521)	(24.816)			(3.621)	(5.942)		
λ_{Hill}	-1.75	110.82	0.00	0.23	1.59	2.86	0.04	0.06	0.27	35.16	0.09	0.27
	(3.840)	(67.286)			(3.779)	(30.153)			(3.317)	(29.537)		
Group C - Option-Return-Based Measures												
$ADBear$	20.89***	−201.24***	0.44	0.51	15.51***	−109.58***	1.30	1.78	3.66***	−22.52***	0.35	0.39
	(4.947)	(78.491)			(3.055)	(32.255)			(1.484)	(7.778)		
$JUMP$	2.63	−298.81**	0.01	0.08	5.47***	45.19*	0.19	0.15	1.55**	−26.06***	0.05	0.67
	(5.852)	(137.860)			(1.551)	(28.837)			(0.651)	(5.660)		
Group D - Macroeconomic Measures												
LE	0.34	−283.63*	0.00	0.86	-1.81	−170.44***	0.02	0.76	2.41	−61.01***	0.07	1.81
	(7.734)	(126.200)			(7.326)	(23.943)			(5.515)	(15.042)		
$PCOneAll$	38.39***	−383.62***	0.36	2.27	19.36***	−152.37***	0.60	1.79	7.68*	−77.29***	0.65	1.76
	(8.253)	(25.363)			(5.879)	(16.306)			(4.770)	(6.328)		
$PCOneOption$	25.56***	−350.24***	0.23	1.71	9.85*	−163.89***	0.29	1.70	2.33	−69.30***	0.31	0.59
	(8.048)	(24.938)			(5.506)	(26.526)			(4.512)	(11.117)		
$PCOneStReturn$	28.90***	−365.28***	0.33	2.26	15.59***	−127.52***	0.58	1.67	6.22**	−46.68***	0.49	2.06
	(5.493)	(49.459)			(4.042)	(17.046)			(3.229)	(4.089)		
$PCOneOpReturn$	15.01***	−337.43***	0.23	0.44	13.40***	−87.56**	1.01	0.80	3.34***	−26.93***	0.28	0.70
	(5.502)	(117.600)			(2.612)	(45.750)			(1.184)	(8.059)		
Controls	Yes				Yes				Yes			

Table XII Multiple Tail Return Predictability

This table presents the coefficients from a return predictability regression. We perform multiple regressions of the market excess returns on lagged tail risk measures, while separating crash and non-crash periods:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot TRM_t \cdot D_{t+\Delta t} + d \cdot D_{t+\Delta t} + e \cdot Controls_t + \epsilon_{t+\Delta t},$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is a vector of the current observations of the tail risk measures. $D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use three different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), and (iii) one-month (*Monthly*). For each forecast horizon, we first perform variable selection based on the PcGets algorithm. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily				Weekly				Monthly			
	b	c	$R^2(b)$	$R^2(c)$	b	c	$R^2(b)$	$R^2(c)$	b	c	$R^2(b)$	$R^2(c)$
Group A - Option-Implied Measures												
$BT11Q$	49.53***		0.62		11.53	−104***	0.24	1.06	9.04*	80***	0.38	0.27
	(11.241)				(8.244)	(24.364)			(5.658)	(17.896)		
$BT14Q$						−50***		0.47	−7.31***		0.65	
						(13.879)			(2.550)			
$BTX15prob$		854***		0.55					5.46		0.78	
		(111.802)							(5.004)			
$BTX15Q$	−23.25***		0.13		−22.65***	61**	0.53	0.32	−9.54***		0.54	
	(8.856)				(6.522)	(26.000)			(3.522)			
H_MRI	−7.38**	547***	0.03	1.01		−131**		1.06		35*		1.52
	(3.633)	(109.927)				(52.794)				(19.635)		
RIX		−346***		0.22						−34		2.00
		(55.323)								(20.823)		
TLM		−717***		1.46	14.15*		0.38		6.95	−22	0.55	0.29
		(49.652)			(9.038)				(6.627)	(13.952)		
Group B - Stock-Return-Based Measures												
$BT11P$	15.99***		0.43		5.63**	−55***	0.46	0.14				
	(5.518)				(2.633)	(12.993)						
CJI										−45***		0.44
										(9.402)		
$JumpRisk$		99***		0.13		−47***		0.11		−4.00		1.88
		(28.341)				(10.078)				(9.405)		
$JumpRP$						−102***		0.48		−54***		0.82
						(23.582)				(11.273)		
λ_{Hill}										−31**		0.63
										(10.169)		
Group C - Option-Return-Based Measures												
$ADBear$					11.18***		0.88					
					(3.052)							
$JUMP$		−208**		0.09		115***		0.21		−7.00		0.21
		(74.195)				(17.097)				(5.171)		
Group D - Macroeconomic Measures												
LE		163***		0.38		58**		0.30				
		(23.176)				(23.597)						
Controls	Yes				Yes				Yes			

Table XIII Cross-Sectional Return Predictability (Value-Weighted)

This table presents the average annualized percentage excess returns of quintile portfolios sorted on the stock loadings on the different tail risk measures. Each month, we estimate the tail risk loadings (b^i) for each stock based on a rolling historical window:

$$R^i_{t+\Delta t} = a^i + b^i \cdot TRM_t + \epsilon^i_t,$$

$R^i_{t+\Delta t}$ is the excess return of stock i over the period between t and Δt. TRM_t is the current observation of a tail risk measure. We forecast stock returns at the daily frequency and use a window length of one month for all measures available at the daily frequency, and accordingly longer windows for measures available on lower frequencies. Based on their current b^i we then sort the stocks into quintile portfolios and obtain the value-weighted portfolio excess return over the next month. We repeat the entire procedure in the next month. The $High - Low$ portfolio simultaneously buys the stocks in the portfolio with the highest b^i and sells those in the portfolio with the lowest b^i. In parentheses, we report robust Newey and West (1987) standard errors using 22 lags. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Low	(2)	(3)	(4)	*High*	*High − Low*
Group A - Option-Implied Measures						
BT11Q	11.01***	10.01***	9.49***	6.33*	1.53	−9.48***
	(3.305)	(3.260)	(3.021)	(3.584)	(5.806)	(3.445)
BT14Q	5.41	6.98	6.98*	7.73**	6.78*	1.37
	(5.131)	(4.230)	(3.900)	(3.618)	(4.079)	(2.887)
BTX15prob	9.51**	8.16**	7.69**	6.71	3.18	−6.33*
	(4.753)	(3.516)	(3.021)	(4.123)	(5.083)	(3.249)
BTX15Q	4.60	8.27**	9.34***	7.08*	6.95	2.35
	(4.685)	(3.303)	(3.189)	(3.745)	(5.186)	(3.390)
H_MRI	8.99**	8.19**	7.23**	6.74	5.32	−3.68
	(4.536)	(3.905)	(3.610)	(4.120)	(4.869)	(2.586)
RIX	9.04***	7.02**	6.87**	6.63	6.31	−2.73
	(3.466)	(3.535)	(3.182)	(4.158)	(6.253)	(3.796)
TLM	9.16**	9.30***	7.83**	7.62**	2.66	−6.51*
	(3.973)	(3.111)	(3.350)	(3.683)	(5.725)	(3.338)
Group B - Stock-Return-Based Measures						
BT11P	9.52**	10.00***	7.34**	5.76	4.14	−5.38*
	(4.577)	(3.080)	(3.431)	(3.917)	(5.672)	(2.773)
CJI	4.01	8.76**	8.39***	8.16**	5.54	1.53
	(4.768)	(3.885)	(3.023)	(3.416)	(4.892)	(2.282)
JumpRisk	8.71**	9.66***	8.38***	6.04	3.54	−5.17*
	(3.837)	(3.097)	(3.007)	(3.956)	(5.782)	(2.999)
JumpRP	9.74***	8.42**	7.35**	7.30*	4.34	−5.39
	(3.538)	(3.458)	(3.119)	(3.756)	(5.669)	(3.397)
λ_{Hill}	11.88**	7.30**	7.44**	6.52*	4.93	−6.95
	(5.390)	(3.625)	(3.586)	(3.902)	(5.084)	(4.726)
Group C - Option-Return-Based Measures						
ADBear	7.56*	8.80**	8.12**	7.85**	3.39	−4.17
	(4.500)	(3.468)	(3.201)	(3.477)	(5.135)	(2.680)
JUMP	7.88**	9.14***	8.29***	8.30**	2.98	−4.90
	(3.902)	(3.073)	(3.110)	(3.475)	(6.521)	(4.157)
Group D - Macroeconomic Measures						
LE	6.63	7.40*	8.33**	7.13*	6.65	0.03
	(4.988)	(3.777)	(3.325)	(3.680)	(4.126)	(2.056)

Table XIV Industrial Production

This table presents the coefficients from a predictive regression for industrial production growth. We perform single regressions of the log growth rate in industrial production (in percentage points) on each tail risk measure, averaged over the previous month:

$$IND_{t+\Delta t} = a + b \cdot \overline{TRM}_t + \epsilon_{t+\Delta t},$$

$IND_{\Delta t}$ is the log change in industrial production over the period Δt. $\overline{TRM}_t$ is the current observation of a tail risk measure, computed as the average of all observations during month t. We use two different forecast horizons Δt: (i) one month (*Monthly*) and (ii) one year (*Annually*). In parentheses, we present robust Newey and West (1987) standard errors with 14 lags. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the R^2s multiplied with 100. "*PCOneAll*", "*PCOnePortfolio*", "*PCOneOption*", and "*PCOneReturn*" denote the first principal components of all measures, the option-implied, stock-return-based and option-return-based tail risk measures, respectively. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Monthly	R^2	*Annually*	R^2
Group A - Option-Implied Measures				
BT11Q	-0.24^{**}	15.75	-0.60^{**}	12.09
	(0.063)		(0.423)	
BT14Q	-0.12	6.36	0.11	10.44
	(0.090)		(0.810)	
BTX15prob	-0.15^{*}	8.09	-0.72	12.84
	(0.100)		(1.421)	
BTX15Q	-0.16^{*}	8.80	0.06	10.40
	(0.084)		(1.168)	
H_MRI	0.07	4.20	0.55	11.83
	(0.061)		(1.818)	
RIX	-0.18^{**}	10.30	-0.09	10.42
	(0.072)		(1.007)	
TLM	-0.19^{*}	10.99	-0.54	11.74
	(0.095)		(0.932)	
Group A - Option-Implied Measures				
BT11P	-0.22^{***}	12.61	-1.28^{***}	15.70
	(0.064)		(0.534)	
CJI	-0.04	3.53	-0.34	10.94
	(0.118)		(1.065)	
JumpRisk	-0.18^{**}	9.74	-1.42^{***}	19.92
	(0.070)		(1.576)	
JumpRP	-0.14^{*}	7.65	-0.78	13.23
	(0.090)		(1.705)	
λ_{Hill}	0.07	4.16	0.80^{*}	13.43
	(0.082)		(1.132)	
Group A - Option-Implied Measures				
ADBear	-0.06^{**}	3.93	-0.88^{***}	14.04
	(0.026)		(0.370)	
JUMP	0.02	3.24	-0.30^{*}	10.81
	(0.030)		(0.264)	
Group A - Option-Implied Measures				
LE	-0.21^{**}	12.03	-0.61	11.43
	(0.085)		(3.385)	
PCOneAll	-0.21^{**}	11.33	-0.65^{*}	9.55
	(0.087)		(1.125)	
PCOneOption	-0.20^{**}	11.99	-0.39	11.09
	(0.088)		(0.434)	
PCOneStReturn	-0.19^{**}	10.23	-1.21^{**}	14.76
	(0.094)		(1.330)	
PCOneOpReturn	0.02	3.22	0.68^{***}	12.58
	(0.028)		(0.339)	

Measuring Tail Risk

Online Appendix

JEL classification: G12, C58, G17, G10

Keywords: Tail Risk, Return Forecasting, Tail Event Forecasting

OA1. Tail Risk Measures

In this section, we describe the tail risk measures in more detail. For further information, we refer the reader to the original papers.

A. Option-Implied Measures

Unless explicitly stated otherwise, for all option-implied measures we follow Bollerslev and Todorov (2011b) and use the options with the shortest maturity available, but with at least 8 days to expiration.

BT11Q Bollerslev and Todorov (2011b) construct a measure of tail risk perceived by investors that is based on close-to-maturity deep out-of-the-money options. They use the insights of the quadratic variation to decompose the volatility into two separate parts in a model-free fashion. To isolate extreme tail risks, they use only deep out-of-the-money options. Only a rare event will be large enough to affect the prices of these derivatives significantly. Bollerslev and Todorov (2011b) use the following definition of the price of a call and put ($C_t(K)$, $P_t(K)$):

$$e^{r(t,T]}P_t(K) \approx \int_t^T \mathbb{E}_t^{\mathbb{Q}}\Big(\int_{\mathbb{R}} 1_{F_{s-}>K} max(0, K - F_{s-}e^x)v_S^{\mathbb{Q}}(dx)\Big)ds,$$

to construct the model-free risk-neutral jump tail measures:

$$LT_t^{\mathbb{Q}}(k) \equiv \frac{1}{T-t}\int_t^T \int_{\mathbb{R}} max(0, e^k - e^x)\mathbb{E}_t^{\mathbb{Q}}(v_S^{\mathbb{Q}}(dx))ds \approx \frac{e^{r(t,T]}P_t(K)}{(T-t)F_{t-}}. \quad \text{(OA1)}$$

We use the approximation above for the calculation of our tail risk measure. The log-moneyness is $k = log(K/F_{t-})$. K is the option's strike price and F_{t-} is the futures price for the aggregate market portfolio. $T - t$ denotes the time-to-maturity as a fraction of a year. As in Bollerslev and Todorov (2011b), we interpolate the option price to the desired moneyness levels, here 0.9, using Black and Scholes (1973) implied volatilities.

BT14Q, ***BTX15Q***, and ***BTX15prob*** Bollerslev and Todorov (2014) and Bollerslev et al. (2015) construct a tail risk estimate using the information from the entire panel of available short-maturity options. The option price is $O_{t,\tau}(k)$ at time t with time-to-maturity τ, price X_t and the log-moneyness is $k = log(K/F_{t-,\tau})$. Bollerslev and Todorov (2011a) show that the jump intensity can be formulated in the following way, with a time-varying shape parameter α^-:

$$\frac{e^{r_{t,\tau}}O_{t,\tau}(k)}{F_{t-,\tau}} \approx \frac{\tau\phi_t^+ e^{k(1-\alpha_t^-)}}{\alpha_t^-(\alpha_t^- - 1)}, \text{ if } k < 0,$$

for the risk-free rate $e^{r_{t,\tau}}$ over period $[t,t+\tau]$. In combination with the extreme-value approximation, Bollerslev and Todorov (2014) follow that the level shift parameter $\phi_t^\pm$ can be purged from the ratio of logarithmic prices, if options with the same time-to-maturity, but different levels of moneyness $(k_1 < k_2)$, are considered:

$$\hat{\alpha_t}^- = \underset{\alpha_t^-}{\operatorname{argmin}} \frac{1}{N_t^-}\sum_{i=2}^{N_t^-} g\left(\frac{log\left(\frac{O_{t,\tau_t}(k_{t,i})}{O_{t,\tau}(k_{t,i-1})}\right)}{k_{t,i} - k_{t,i-1}} - (1 + \alpha_t^-)\right).$$

Thus, Bollerslev and Todorov (2014) conclude that the tail shape α^- can be estimated from an increasing span of options over either an increasing range of strikes or an increasing sample span. This method imposes only a parametric structure on the jump intensity, not on the level shift estimates (ϕ^-). Because option data is not continuously available, Bollerslev and Todorov (2014) pool the parameters and obtain weekly, monthly, or annual tail shape parameter estimates.

Because of the noise of the parameters α_t^- Bollerslev and Todorov (2014) propose the following parametric model, to smooth the estimates:

$$\alpha_j^- = \beta_0^- + \beta_1^-\alpha_{j-1}^- + \beta_2^- log(1 + QV^c_{\tau_{j-1},\tau_j}) + \beta_3^- log(1 + QV^d_{\tau_{j-1},\tau_t}) + \epsilon_j^- \tag{OA2}$$

where $j = 1, ..., J$ refers to the weeks in the sample and QV to the quadratic variation of the series (the variation that includes both jumps and continuous variation). QV^c refers to the continuous variation in the sample and QV^d to the discontinuous portion of the total variation. Bollerslev and

Todorov (2014) then subsequently estimate the model using the function:

$$\hat{\beta}^{-} = \underset{\beta^{-}}{\text{argmin}} \sum_{j=1}^{J} \sum_{t=\tau_{j-1}}^{\tau_j} \frac{1}{N_t^{-}} \sum_{i=2}^{N_t^{-}}$$
$$g\left(\frac{log(\frac{O_{t,\tau_t}(k_{t,i})}{O_{t,\tau}(k_{t,i-1})})}{k_{t,i}-k_{t,i-1}} - \Big(1-\beta_0^{-}-\beta_1^{-}\hat{\alpha}_{j-1}^{-}-\beta_2^{-}log(1+QV^{c}_{(\tau_{j-1},\tau_j]})-\beta_3^{-}log(1+QV^{d}_{(\tau_{j-1},\tau_j]})\Big)\right),$$

and omit all variables that are insignificant at the 5% level. QV is calculated using 5-minute prices. To estimate the tail of the distribution, not only the tail shape $\alpha_t^{\pm}$ needs to be estimated, but also the level shift $\phi_t^{\pm}$. After the estimation of $\alpha_t^{\pm}$, $\phi_t^{\pm}$ can be estimated in a second step:

$$\hat{\phi}_t^{-} = \underset{\phi_t^{-}}{\text{argmin}} = \frac{1}{N_t^{-}} \sum_{i=1}^{N_t^{-}} \left| log\left(\frac{e^{r_{t,\tau}}O_{t,\tau}(k_{t,i})}{\tau F_{t,\tau}}\right) - (1+\hat{\alpha}_t^{-})k_{t,i} + log(\hat{\alpha}_t^{-}+1) + log(\hat{\alpha}_t^{-}) - log(\phi^{-})\right|.$$

In turn, the jump intensity process is characterized by: $v_t^{\mathbb{Q}} = (\phi_t^{+}e^{-\alpha_t^{+}x}1_{x>0} + \phi^{-}e^{-\alpha_t^{-}|x|}1_{x<0})$, and can be estimated via:

$$LJV^{\mathbb{Q}}_{t,t+\tau} = \frac{\tau\phi_t^{-}e^{-\alpha_t^{-}|k_t|}(\alpha_t^{-}k_t(\alpha_t^{-}k_t+2)+2)}{(\alpha_t^{-})^3}. \tag{OA3}$$

k_t is a threshold that serves as a cutoff point at each tail, we define it as: $k_t = 10\sigma_{ATM,30d}\sqrt{5/252}$, $\sigma_{ATM,30d}$ is the at-the money 30-day volatility of the interpolated option surface obtained from Option Metrics.

Alternatively, Bollerslev et al. (2015) obtain the estimates for $\hat{\alpha}_t^{-}$ and $\hat{\phi}_t^{-}$ with a non-parametric estimation:

$$\begin{aligned} \hat{\alpha}_t^{-} &= median\left|1-\frac{log\frac{O_{t,\tau}(k_{t,i})}{O_{t,\tau}(k_{t,i-1})}}{k_{t,i}-k_{t,i-1}}\right|, \\ \hat{\phi}_t^{-} &= median\left|log\left(\frac{e^{r_{t,\tau}O_{t,\tau}(k_{t,i})}}{\tau F_{t,\tau}}\right) - (1-\hat{\alpha}_t^{-})k_{t,i} + log(\hat{\alpha}_t^{-}+1) + log(\hat{\alpha}_t^{-})\right|. \end{aligned} \tag{OA4}$$

The current tail estimates are measures of return variation expected by the market in the left or the right tail of the distribution. The threshold depends on the current volatility. It might also be useful to consider a constant threshold and obtain the probability, similar to the commonly used Value-at-Risk (VaR). For a probability measure that captures the probability of a 10% crash over the next week, the equation can be rearranged in the following way:

$$LeftProb_t = 100\hat{\phi}_t^- \frac{e^{-\hat{\alpha}_t^- |k_t|}}{\hat{\alpha}_t^-}. \tag{OA5}$$

In our analysis we will use the probability measure and the left tail risk measure, estimated non-parametrically and the parametrically smoothed and estimated left tail risk measure, both of which have a correlation of just 75%.

H_MRI Gormsen and Jensen (2020) develop a measure of higher-moment risk, based on the out-of the money put and call options. They use the inference techniques developed by Breeden and Litzenberger (1978) and Bakshi, Kapadia, and Madan (2003) to infer the ex-ante moments. The moments are estimated from out-of the money put and call options, using the following representation:

$$\begin{aligned}
E_t[R_{t,T}^n] &= \frac{(R_{t,T}^f)^{n+\gamma} + R_{t,T}^f \left[\sum_{i=1}^N \frac{(\gamma+n)(\gamma+n-1)}{S^{\gamma+n}} (S_t R_{t,T}^f - F_{t,T} + K_i)^{n+\gamma-2} \Omega_{t,T}(K_i)\Delta K_i\right]}{(R_{t,T}^f)^{\gamma} + R_{t,T}^{\gamma} \left[\sum_{i=1}^N \frac{\gamma(\gamma-1)}{S^{\gamma}} (S_t R_{t,T}^f - R_{t,T} + K_i)^{\gamma-2} \Omega_{t,T}(K_i)\Delta K_i\right]}, \\
\Omega_{t,T} &= \begin{cases} call_{t,T}(K) & \text{if } K \geq F_{t,T}, \\ put_{t,T}(K) & \text{if } K < F_{t,T}, \end{cases} \\
\Delta K_i &= \begin{cases} K_{i+1} - K_i & \text{if } i = 1, \\ K_i - K_{i-1} & \text{if } i = N, \\ \frac{K_{i+1}-K_{i-1}}{2} & \text{else.} \end{cases}
\end{aligned} \tag{OA6}$$

Strike prices K_1, ..., K_N of the N out-of-the money options are in ascending order. The moments are then calculated using the following calculations:

$$
\begin{aligned}
Skewness_{t,T} &= \frac{E_t[R^3_{t,T}] - 3E_t[R_{t,T}]E_t[R^2_{t,T}] + 2E_t[R_{t,T}]^3}{(E_t[R^2_{t,T}] - E_t[R_{t,T}]^2)^{(3/2)}}, \\
Kurtosis_{t,T} &= \frac{E_t[R^4_{t,T}] - 3E_t[R_{t,T}]^4 + 6E_t[R_{t,T}]^2E_t[R^2_{t,T}] - 4E_t[R_{t,T}]E_t[R^3_{t,T}]}{(E_t[R^2_{t,T}] - E_t[R_{t,T}]^2)^2}, \\
Hyperskewness_{t,T} &= \frac{E_t[R^5_{t,T}] + 4E_t[R_{t,T}]^5 + 10E_t[R_{t,T}]^2E_t[R^3_{t,T}]}{(E_t[R^2_{t,T}] - E_t[R_{t,T}]^2)^{(5/2)}} + \\
&\quad \frac{-10E_t[R_{t,T}]^3E_t[R^2_{t,T}] - 5E_t[R_{t,T}]E_t[R^4_{t,T}]}{(E_t[R^2_{t,T}] - E_t[R_{t,T}]^2)^{(5/2)}}, \\
Hyperkurtosis_{t,T} &= \frac{E_t[R^6_{t,T}] - 5E_t[R_{t,T}]^6 + 15E_t[R_{t,T}]^4E_t[R^2_{t,T}]}{(E_t[R^2_{t,T}] - E_t[R_{t,T}]^2)^3} + \\
&\quad \frac{-20E_t[R_{t,T}]^3E_t[R^3_{t,T}] + 15E_t[R_{t,T}]^2E_t[R^4_{t,T}] - 6E_t[R_{t,T}]E_t[R^5_{t,T}]}{(E_t[R^2_{t,T}] - E_t[R_{t,T}]^2)^3}.
\end{aligned}
$$

We linearly interpolate between the times-to-maturity to generate the moments with constant 30-day time-to-maturity. To obtain the higher-moments risk index, we use the first PC of the four measures. The first PC loads positively on the kurtosis measures and negatively on the skewness measures.

RIX Gao et al. (2018) and Gao et al. (2019) construct an index for tail risk concern (RIX). They use two option portfolios to model the expected downside movement of the market. The two portfolios have the following design:

$$
\begin{aligned}
IV^- &\equiv \frac{2e^{r\tau}}{\tau}\int_{K<X_t} \frac{1}{K^2}P(X_t;K,\tau)dK, \\
V^- &\equiv \frac{2e^{r\tau}}{\tau}\int_{K<X_t} \frac{1 - log(K/X_t)}{K^2}P(X_t;K,\tau)dK.
\end{aligned}
\tag{OA7}
$$

Both portfolios differ on how they assign weight to out-of-the money put option prices, V^- assigns relatively larger weight to deeper out-of-the money options. The resulting index is constructed by going long in the portfolio with higher exposure to deep out-of-the money options (V^-) and short the portfolio with a lower exposure (IV^-), resulting in a positive exposure of the portfolio towards jump risk, while being relatively immune to volatility risk. In order to estimate both portfolios, we interpolate the implied volatility of the options with a cubic spline along the moneyness, following

Gao et al. (2018). This way, we generate 2,000 artificial options in a strike range between zero and three times the price of the underlying. For artificial options outside the observed strike range, we constantly extrapolate the implied volatilities. The resulting portfolio can be constructed with the following weights (Gao et al., 2018):

$$RIX^{-} \equiv V^{-} - IV^{-} = \frac{2e^{r\tau}}{\tau} \int_{K<X_t} \frac{log(X_t/K)}{K^2} P(X_t; K, \tau) dK, \tag{OA8}$$

where r is the constant risk-free rate and P is the price of the out-of-the money put option with maturity τ and strike price K. We use a trapezoidal rule to approximate the integrals. Finally, we linearly interpolate between the two times-to-maturity closest to (above and below) 30 days to generate the RIX^{-} with a constant 30-day time-to-maturity.

TLM Vilkov and Xiao (2015) build on Extreme Value Theory (EVT) to estimate the measure of tail risk implied from option prices under the risk-neutral measure. According to EVT, the price $P_t(K)$ of an out-of-the-money put option can be calculated using the rules of conditional volatilities.

Combining the equations of the EVT, Vilkov and Xiao (2015) rewrite the price of a further out-of-the money option as a function of an option that is closer at-the-money.

$$P_t(K_1) = P_t(K) \left(1 + \xi \times \frac{K - K_1}{\beta(K)}\right)^{1-1/\xi}. \tag{OA9}$$

Vilkov and Xiao (2015) use the above equation to price deep out-of-the-money puts relative to a boundary put. Then, they compare the theoretically obtained price with the empirical price and infer the parameters $\beta(K)$ and ξ by minimizing the pricing errors. Finally, Vilkov and Xiao (2015) estimate the tail loss measure on a given threshold via $TLM = \frac{\beta(K)}{1-\xi}$, which they estimate as follows:

$$
\begin{aligned}
K_{0,\tau} &= S_t \left(1 - 2\frac{\widehat{VIX}_t/100}{\sqrt{12}}\right), \\
\widehat{VIX}_t &= \frac{1}{63}\sum_{i=0}^{62} VIX_{t-i}, \\
P^*_{i,t} &= P^*_{0,t}\left[\frac{\xi_t}{\beta_t}(K_{0,t} - K_{i,t}) + 1\right]^{1-1/\xi_t}, \\
\{\xi_t, \beta_t\} &= \operatorname{argmin} \sum_{i=0}^{n-1} \left|\frac{P_{i,t} - P^*_{i,t}}{P^*_{i,t}}\right|, \\
TLM &= \frac{\beta_t}{1 - \xi_t}.
\end{aligned}
$$

B. Stock-Return-Based Measures

BT11P Bollerslev and Todorov (2011a) use a threshold estimator. To identify the presence of jumps, they use the bipower variation (BV_t) proposed by Barndorff-Nielsen and Shephard (2004, 2006) and the realized variation (RV_t). The bipower measures should only identify the continuous part of the variation and an exceedance should clearly indicate the jumps, Bollerslev and Todorov (2011a) use for the threshold:

$$\alpha_t = 4\sqrt{BV_t \wedge RV_t}.$$

To avoid false positives, Bollerslev and Todorov (2011a) use a time-of-day factor (TOD), to adjust alpha for the intraday pattern of volatility. Bollerslev and Todorov (2011b) say any $\alpha > 0$ and $\omega \in (0, 0.5)$ would work, but they fix ω to 0.49. α, however, is chosen as displayed above. They define the TOD factor in the following way:

$$
\begin{aligned}
TOD_i =& NOI_i \frac{\sum_{t=1}^{N}(p_{t-1+\pi_t+i\Delta_{n,t}} - p_{t-1+\pi_t+(i-1)\Delta_{n,t}})^2 1_{|p_{t-1+\pi_t+i\Delta_{n,t}} - p_{t-1+\pi_t+(i-1)\Delta_{n,t}}| \leq \bar{\alpha}\Delta_n^{0.49}}}{\sum_{t=1}^{N}\sum_{i=1}^{M}(p_{t-1+\pi_t+i\Delta_{n,t}} - p_{t-1+\pi_t+(i-1)\Delta_{n,t}})^2}, \text{ where} \\
NOI_i =& \frac{\sum_{t=1}^{N}\sum_{i=1}^{n-1} 1_{|p_{t-1+\pi_t+i\Delta_{n,t}} - p_{t-1+\pi_t+(i-1)\Delta_{n,t}}| \leq \bar{\alpha}\Delta_n^{0.49}}}{\sum_{t=1}^{N} 1_{|p_{t-1+\pi_t+i\Delta_{n,t}} - p_{t-1+\pi_t+(i-1)\Delta_{n,t}}| \leq \bar{\alpha}\Delta_n^{0.49}}}, \text{ and} \\
\bar{\alpha} =& 4\sqrt{\frac{\pi}{2}}\sqrt{\frac{1}{N}\sum_{t=1}^{N}\sum_{i=2}^{n-1}|p_{t-1+\pi_t+i\Delta_{n,t}} - p_{t-1+\pi_t+(i-1)\Delta_{n,t}}||p_{t-1+\pi_t+(i-1)\Delta_{n,t}} - p_{t-1+\pi_t+(i-2)\Delta_{n,t}}|}.
\end{aligned}
$$

Thus, the intraday α is:

$$\alpha_{t,i} = 4\sqrt{BV_t \wedge RV_t} \times TOD_i \times \Delta_n^{0.49}. \tag{OA10}$$

Bollerslev and Todorov (2011a) define the following parameter vector that can be estimated: $\theta \equiv (\sigma^-, \xi^-, k_0^- \bar{v}_\psi^-(\varrho_T), k_1^- \bar{v}_\psi^-(\varrho_T))$. The estimation is based on the scores associated with the log-likelihood function of the generalized Pareto distribution. Specifically Bollerslev and Todorov (2011a) estimate the following equations:

$$\begin{aligned}
\frac{1}{N}\sum_{t=1}^{N}\sum_{j=1}^{n-1}\phi_i^-(\psi^-(\Delta_j^{n,t}p) - tr^-)1_{\psi^-(\Delta_j^{n,t}p)>tr^-} &= 0,\\
\frac{1}{N}\sum_{t=1}^{N}\sum_{j=1}^{n-1}1_{\psi^-(\Delta_j^{n,t}p)>tr^-} - (1-\phi)k_0^-\bar{v}_\psi^-(\varrho_T) - k_1^-\bar{v}_\psi^-(\varrho_T))CV_t &= 0,\\
\frac{1}{N}\sum_{t=2}^{N}\left(\sum_{j=1}^{n-1}1_{\psi^-(\Delta_j^{n,t}p)>tr^-} - (1-\phi)k_0^-\bar{v}_\psi^-(\varrho_T) - k_1^-\bar{v}_\psi^-(\varrho_T))CV_t\right)CV_{t-1} &= 0,\\
\text{and } (1-\phi)\frac{1}{N}\sum_{t=1}^{N}(p_{t+\phi_t} - p_t)^2 - \phi\frac{1}{N}\sum_{t=1}^{N}RV_t &= 0.
\end{aligned}$$

$n = \frac{1}{\Delta_n}$ is the number of high-frequency price observations over one day. $\Delta_i^{n,t}p := p_{t+i\delta_n} - p_{t+(i-1)\Delta_n}$ refers to the corresponding price increments over one day. Furthermore Bollerslev and Todorov (2011a) define the tail parameter as tr^-, such that it corresponds in log-prices of 0.6%. For tr^- this implied $e^{0.006} \approx 1.006$ for the left tails, respectively.

CJI Christoffersen et al. (2012) fit a parametric model. To investigate the tail risk dynamics using daily returns, the authors propose four nested models; we present the most general and best performing model, the DVSDJ model (dynamic volatility with separate dynamic jumps). The model can be estimated using only return data, or both option and return data. They propose the following model specifications:

$$\begin{aligned}
h_{z,t+1} &= w_z + b_z h_{z,t} + \frac{a_z}{h_{z,t}}(z_t - c_z h_{z,t})^2 + d_z(y_t - e_z)^2,\\
h_{y,t+1} &= w_y + b_y h_{y,t} + \frac{a_y}{h_{z,t}}(z_t - c_y h_{z,t})^2 + d_y(y_t - e_y)^2.
\end{aligned} \tag{OA11}$$

$h_{z,t+1}$ is the return innovation for the market price of risk of the normal component, while $h_{y,t+1}$ is the return innovation for the market price of risk of the jump component. In order to obtain the unobservable measures, Christoffersen et al. (2012) propose a filtering technique for the returns to obtain the number of jumps (n_t), the normal component of the return (z_t) and the jump component of the return (y_t). Then, the variance of the normal, and the jump component can be determined. First, Christoffersen et al. (2012) use Bayes' rule to filter the density:

$$Pr_t(n_t = j) \equiv Pr_{t-1}(n_t = j|x_t) = \frac{f_{t-1}(x_t|n_t = j)Pr_{t-1}(n_t = j)}{f_{t-1}(x_t)}, \tag{OA12}$$

where

$$\begin{aligned}
f_t(x_{t+1}|n_{t+1} = j) &= \frac{1}{\sqrt{2\pi(h_{z,t+1} + j\delta^2)}} exp\left(-\frac{(x_{t+1} - \mu_{t+1} - j\theta)^2}{2(h_{z,t+1} + j\delta^2)}\right), \\
Pr_t(n_{t+1} = j) &= \frac{(h_{y,t+1})^j}{j!} exp(-h_{y,t+1}), \\
f_t(x_{t+1}) &= \sum_{j=0}^{\infty} f_t(x_{t+1}|n_{t+1} = j)Pr_t(n_{t+1} = j).
\end{aligned}$$

$Pr_t(n_t = j)$ is the ex-post inference on n_t. Multiplying the density function by the amount of jumps results in the filtered number of jumps: $\tilde{n}_t = \sum_{j=0}^{\infty} jPr_t(n_t = j)$. To solve the ex-post filtration on the normal component, Christoffersen et al. (2012) filter the expectation of z_t. If the return and the number of jumps are known, Christoffersen et al. (2012) define z_t in the following way:

$$z_t(x_t, n_t = j) = \sqrt{\frac{\tilde{h}_{z,t}}{\tilde{h}_{z,t} + j\delta^2}}(x_t - \mu_t - j\theta). \tag{OA13}$$

$\tilde{h}_{z,t}$ is the filtered $h_{z,t}$, μ_t is the first conditional return moment; Christoffersen et al. (2012) define it as follows: $\mu_t = r + (\lambda_z - 0.5)h_{z,t} + (\lambda_y - \xi)$. The expectation can be solved via the following summation: $\tilde{z}_t = E_t[z_t] = \sum_{j=0}^{\infty} z_t(x_t, n_t = j)Pr_t(z_t, n_t = j)$, where $Pr_t(z_t, n_t = j) \equiv Pr_{t-1}(z_t, n_t = j|x_t) \propto Pr_{t-1}(z_t|x_t, n_t = j)Pr_t(n_t = j)$.

$Pr_{t-1}(z_t|x_t, n_t = j) = Pr_{t-1}(x_t, n_t = j|x_t, n_t = j)\sqrt{\frac{\tilde{h}_{z,t}}{\tilde{h}_{z,t}+j\delta^2}}$, the first term on the right-hand-side of this equation is one, using this equation, and Equation (OA13) to be:

$$
\begin{aligned}
\tilde{z}_t &= \sum_{j=0}^{\infty} z_t(x_t, n_t = j) Pr_{t-1}(z_t | x_t, n_t = j) Pr_t(n_t = j) \\
&= \sum_{j=0}^{\infty} \frac{\tilde{h}_{z,t}}{\tilde{h}_{z,t} + j\delta^2} (x_t - \mu_t - j\theta) Pr_t(n_t = j).
\end{aligned} \tag{OA14}
$$

From $\tilde{z}_t$ Christoffersen et al. (2012) can directly infer the filtered jump innovation $\tilde{y}_t$. It is given by $\tilde{y}_t = x_t - \mu_t - \tilde{z}_t$. With the two variables, the filtered variance and the jump intensity for the next period can be computed:

$$
\begin{aligned}
\tilde{h}_{z,t+1} &= w_z + b_z \tilde{h}_{z,t} + \frac{a_z}{\tilde{h}_{z,t}} (\tilde{z}_t - c_z \tilde{h}_{z,t})^2 + d_z(\tilde{y}_t - e_z)^2, \\
\tilde{h}_{y,t+1} &= w_y + b_y \tilde{h}_{y,t} + \frac{a_y}{\tilde{h}_{z,t}} (\tilde{z}_t - c_y \tilde{h}_{z,t})^2 + d_y(\tilde{y}_t - e_y)^2.
\end{aligned} \tag{OA15}
$$

For the likelihood at time t=0, Christoffersen et al. (2012) assume that the time-series is equal to the mean of the filtered time-series from the prior iteration. With the filtered data, Christoffersen et al. (2012) conduct the following optimization via maximum likelihood:

$$
L_{returns} = \sum_{t=1}^{\tau-1} log(f_t(x_{t+1})) = \sum_{t=1}^{\tau-1} log \left(\sum_{j=0}^{\infty} f_t(x_{t+1|n_{t+1}=j}) Pr_t(n_{t+1} = j) \right), \tag{OA16}
$$

with

$$
\begin{aligned}
f_t(x_{t+1} | n_{t+1} = j) &= \frac{1}{\sqrt{2\pi(\tilde{h}_{z,t+1} + j\delta^2)}} exp\left(-\frac{(x_{t+1} - \mu_{t+1} - j\theta)^2}{2(\tilde{h}_{z,t+1} + j\delta^2)} \right), \\
Pr_t(n_{t+1} = j) &= \frac{(\tilde{h}_{y,t+1})^j}{j!} exp(-\tilde{h}_{y,t+1}).
\end{aligned}
$$

JumpRisk and ***JumpRP*** Maheu et al. (2013) estimate a stochastic jump model to calculate the time-varying jump risk from the daily returns of a time-series. They use a utility-based framework to achieve this. Maheu et al. (2013) assume that the innovation in the return process stems from two stochastic processes, $\epsilon_{1,t} = \sigma_t * N(0,1)$ and $\epsilon_{2,t} = \sum_{K=1}^{n+1} N(\theta, \delta^2) - \theta\lambda_t$. To obtain the measure Maheu et al. (2013) estimate the following system of equations:

$$P(n_t = j|\Phi_{t-1}) = \frac{e^{\lambda_t}\lambda_t^j}{j!},\ j = 0, 1, 2, ...,$$

$$\lambda_t = E[n_t|\Phi_{t-1}] = \lambda_0 + \rho\lambda_{t-1} + \gamma\xi_{t-1},$$

$$\xi_{t-1} = \sum_{j=0}^{\infty} jP(n_{t-1} = j|\Phi_{t-1}) - \lambda_{t-1},$$

$$E[\lambda_t] = \frac{\lambda_0}{1-\rho},$$

$$E[\lambda_{t+i}|\Phi_{t-1}] = \begin{cases} \lambda_t & i = 0. \\ \lambda_0(1 + p + ... + p^{i-1}) + p\lambda_t & i \geq 1. \end{cases}$$

$$\lambda_t = \lambda_0 + (\rho - \gamma)\lambda_{t-1} + \gamma E[n_{t-1}|\Phi_{t-1}].$$

As a start value for λ_t Maheu et al. (2013) use $E[\lambda_t]$, for ξ_1 we choose 0. Maheu et al. (2013) calculate first the following result:

$$P(n_{t+1} = j|\Phi_{t+1}, \theta) = \frac{f(r_{t+1}|n_{t+1} = j, \Phi_t, \theta)P(n_{t+1} = j|\Phi_t, \theta)}{f(r_{t+1}|\Phi_t, \theta)}, \tag{OA17}$$

$$f(r_{t+1}|n_{t+1} = j, \Phi_t, \theta) = \frac{1}{\sqrt{2\pi(\sigma_t^2 + j\delta^2)}} e^{-0.5\frac{(r_{t+1} - m_t - \rho_1(r_t - m_{t-1}) - \rho_2(r_{t-1} - m_{t-2}) - (j - \lambda_t)\theta)^2}{\sigma_t^2 + j\delta^2}},$$

$$P(n_{t+1} = j|\Phi_t, \theta) = \frac{e^{-\lambda_{t-1}^j}\lambda_{t-1}^j}{j!},\ j = 0, 1, 2, ...,$$

$$f(r_{t+1}|\Phi_t, \theta) = \sum_{j=0}^{\infty} f(r_{t+1}|n_{t+1} = j, \Phi_t)P(n_{t+1} = j|\Phi_t),$$

$$\lambda_t = \lambda_0 + \rho\lambda_{t-1} + \gamma\xi t - 1,$$

$$E[\lambda_t] = \frac{\lambda_0}{1-\rho}.$$

As Maheu et al. (2013) state, risk premia in their model behave opposite to the current state of volatility and jump risk. This often leads to low estimations in crisis periods. Thus, we conduct the regressions with inverse jump risk premia. We use two measures from this estimation. We use the risk of a jump (λ_t) and we use the jump risk premium, which is calculated using the first derivative of the equity risk premium (m_t) with respect to λ_t.

$\boldsymbol{\lambda_{Hill}}$ Kelly and Jiang (2014) assume that returns obey the dynamic power law structure for equity returns. In their specification, the tail distribution obeys a potentially time-varying power law. In order to improve upon a main obstacle, the low sample size for individual returns, Kelly and Jiang (2014) exploit the information in the cross-section of stock returns. Thus, they assume that all individual assets have tail risks that are governed by a single process. Kelly and Jiang (2014) apply the power law of Hill (1975). The estimator is defined for a pooled cross-section as follows:

$$\lambda_t^{Hill} = \frac{1}{E_t} \sum_{k=1}^{E_t} log \left(\frac{X_{k,t}}{u_t} \right). \tag{OA18}$$

u_t is the extreme-value threshold in month t. E_t is the total number of exceedances of u_t in a month, all cross-sectional returns in this month are considered; this is without loss of generality, because this estimator does not consider any differences in the tails for each company. u_t is chosen by the econometrician to define where the tail of the distribution begins. Kelly and Jiang (2014) define the threshold as the fifth percentile of the cross-section in the sample. The estimator only considers exceedances of u_t for the power law. Kelly and Jiang (2014) refer to this exponent as tail risk. To remove dependencies in the returns of the observed returns, Kelly and Jiang (2014) use the residuals from a regression with the common return factors of Fama and French (1993).

C. Option-Return-Based Measures

ADBear Lu and Murray (2019) construct another measure that uses option portfolios. They create a portfolio that yields a positive payoff of \$1 when the S&P 500 is below a certain threshold K_2. To create a tradeable position of this portfolio, they take a short position in a put option with strike price $K_1 > K_2$ and a short position in a put option with strike price K_2. Then they scale the positions by $K_1 - K_2$ to achieve the desired payoff. This generates a payoff that is \$1 below K_2 and is linearly decreasing between K_2 and K_1. The price of the portfolio is then the following:

$$P_{\text{AD Bear}} = \frac{P(K_1) - P(K_2)}{K_1 - K_2}\ . \tag{OA19}$$

Lu and Murray (2019) define K_2 to be 1.5 standard deviations below the S&P 500 index forward price. This threshold is chosen based with the objective of capturing the pricing of the extreme left tails of the index, while avoiding the noise of the extreme tails. K_1 is chosen to be 0.5 standard deviations above K_2. The standard deviation is the level of the VIX index divided by 100, multiplied by the square root of the time to maturity. In order to create a price for the desired out-of-the-money put option, Lu and Murray (2019) calculate the price to be the volume weighted average price of the put options within a 0.25 standard deviation range of the desired targeted strike price. This leads to the following specification:

$$\begin{aligned} P(K_1) &= \sum_{K \in [Fe^{-1.25\frac{VIX}{100}\sqrt{\tau}}, Fe^{-0.75\frac{VIX}{100}\sqrt{\tau}}]} P(K)w(K), \\ P(K_2) &= \sum_{K \in [Fe^{-1.75\frac{VIX}{100}\sqrt{\tau}}, Fe^{-1.25\frac{VIX}{100}\sqrt{\tau}}]} P(K)w(K). \end{aligned} \tag{OA20}$$

For liquidity reasons, Lu and Murray (2019) consider only one-month options, which are options that expire in the next month. The portfolio is held for the next five trading days, but the portfolio is constructed daily. In addition, they subtract the five-day risk-free rate from the returns. As a result Lu and Murray (2019) have five-day overlapping $ADBear$ portfolio excess returns, which we use as our jump risk measure.

$\boldsymbol{JUMP}$ Cremers et al. (2015) construct factors for volatility and jump risk with delta-neutral at-the-money straddles. They construct delta-neutral at-the-money straddles to create portfolios that mimic volatility or jump risk. The straddles have large vegas as well as high gammas.[1] Cremers et al. (2015) create two portfolios, one with exposure to vega and another that is only exposed to gamma risk. These portfolios capture exclusively volatility or jump risk. To create gamma or vega neutral straddles, Cremers et al. (2015) use the fact that the gamma of an option is decreasing with increasing time to maturity, while vega is increasing with increasing time to maturity. They are able to create both strategies with long/short portfolios involving market-neutral straddles with different maturities.

[1] Thus, a high sensitivity towards volatility and jumps, respectively.

They construct a zero-beta straddle:

$$x_{MN} = \theta x_c + (1-\theta)x_p,$$
$$0 = \theta\beta_c + (1-\theta)\beta_p.$$

x_{MN} is the return of a market-neutral straddle , x_c is the return of a call, x_p is the return of a put. β_c and β_p are the market betas of the call and put options. To calculate the sensitivities Cremers et al. (2015) use the Black and Scholes (1973) option pricing formula.

Cremers et al. (2015) create the following two portfolios: A jump risk factor-mimicking portfolio (JUMP) is a market-neutral, vega-neutral, and gamma-positive strategy, where the time-to maturity $T_2 > T_1$. Thus, they use (i) a long position in one market-neutral at-the-money straddle with maturity T_1 and (ii) a short position in y market-neutral at-the-money straddles with maturity T_2. y is chosen to create a vega-neutral portfolio. To construct the short-dated straddles Cremers et al. (2015) use the option pair that is being closest at-the-money. For short-dated straddles they choose the options that expire in the next calendar month, for long-dated options they choose options that expire in the calendar month that follows the next month. The strategy is re-balanced daily. For our analysis, we use the returns of the gamma-positive, vega- and market-neutral $JUMP$ strategy.

D. Macroeconomic Measures

LE Adrian et al. (2019) develop a measure that infers tail risk for GDP growth from an index of financial conditions. For this purpose, they use the National Financial Conditions Index (NFCI) from Brave and Butters (2012) provided by the Chicago FED.[2] They use quantile regressions to empirically estimate the quantiles of the distribution. They use this to fit a skewed t-distribution, to infer the entire distribution. The authors minimize the squared error between the estimation from the quantile distribution, based on the parameters of the t-distribution and the skewed t-distribution every quarter. They argue that financial conditions can account for a proportion of GDP growth; especially in the left tail, tightening financial conditions leads to downside risks in

[2] `https://www.chicagofed.org/`.

GDP growth. Adrian et al. (2019) fit the following quantile regression:

$$\hat{Q}_{y_{t+\tau}|x_t}(\tau|x_t) = x_t\hat{\beta}_\tau. \tag{OA21}$$

They use the estimates, to create a quantile function (the inverse cumulative distribution function) and fit the quantile function to the skewed t-distribution to recover a probability density function:

$$f(y;\mu,\sigma,\alpha,v) = \frac{2}{\sigma}t\left(\frac{y-\mu}{\sigma};v\right)T\left(\alpha\frac{y-\mu}{\sigma}\sqrt{\frac{v+1}{v+\left(\frac{y-\mu}{\sigma}\right)^2}};v+1\right), \tag{OA22}$$

where $t(\bullet)$ is the probability density function (PDF) of the Student t-distribution and $T(\bullet)$ is the cumulative distribution function (CDF) of the Student t-distribution. μ is the location, σ is the scale, v is the fatness, and α is the shape. For each quarter, following Adrian et al. (2019), the four parameters are chosen to minimize the distance between the estimated quantile function $\hat{Q}_{y_{t+\tau}|x_t}(\tau|x_t)$ and the quantile function of the skewed t-distribution $F^{-1}(\tau;\mu,\sigma,\alpha,v)$ at the 5, 25, 75, and 95 quantiles:

$$\{\hat{\mu}_{t+\tau},\hat{\sigma}_{t+\tau},\hat{\alpha}_{t+\tau},\hat{v}_{t+\tau}\} = \underset{\mu,\sigma,\alpha,v}{\operatorname{argmin}}\sum_\tau(\hat{Q}_{y_{t+\tau}|x_t}(\tau|x_t) - F^{-1}(\tau;\mu,\sigma,\alpha,v))^2. \tag{OA23}$$

We retain two measures from this estimation for each tail: the expected shortfall and the left entropy. While the expected shortfall is the expectation of the worst outcomes of economic growth, the left entropy describes the left-skewedness of the distribution.

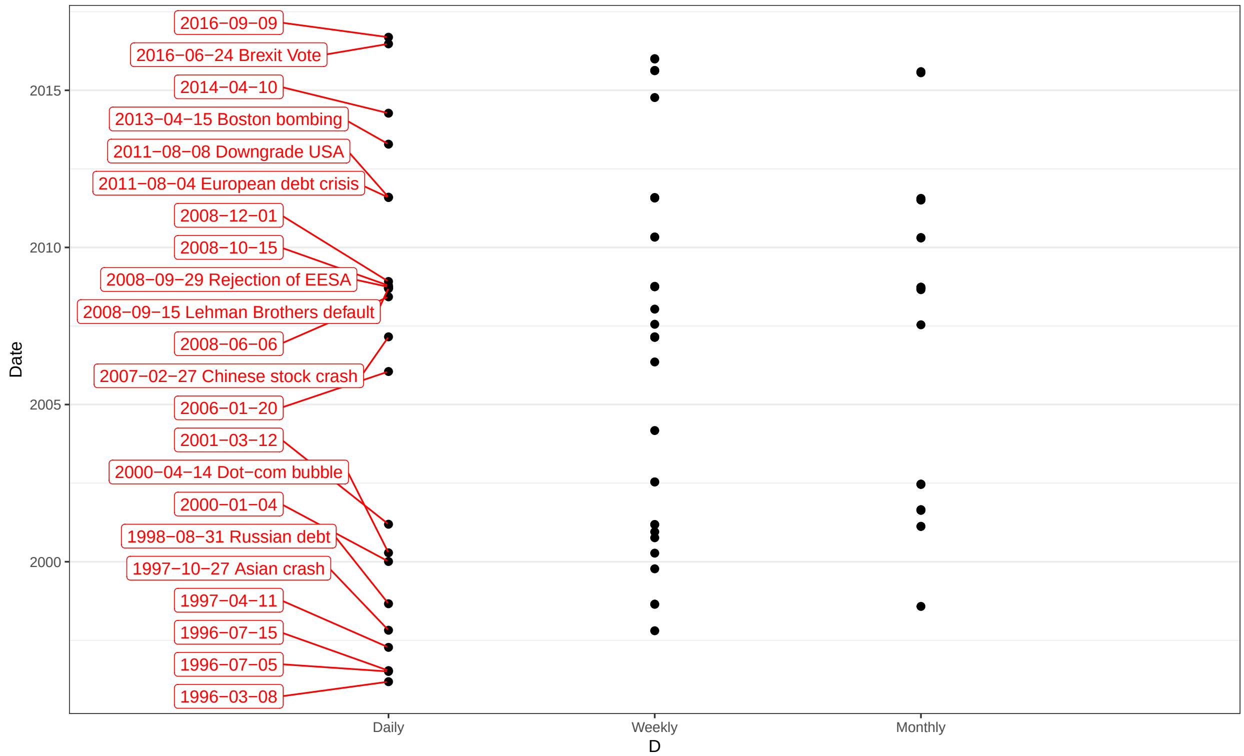

Figure A1. This figure displays the realized tail events ($D_{t+\Delta t}$). The points illustrate the times of the tail event realizations at the daily, weekly, and monthly horizons. For the daily frequency, in red we display the exact dates and, in cased where the events are clearly linked to certain events, we also indicate these events. The numbers of observed left-tail events are 24 at the daily horizon, 37 at the weekly horizon, and 46 at the monthly horizon. Weekly and monthly tail events are clustered due to the use of overlapping return windows.

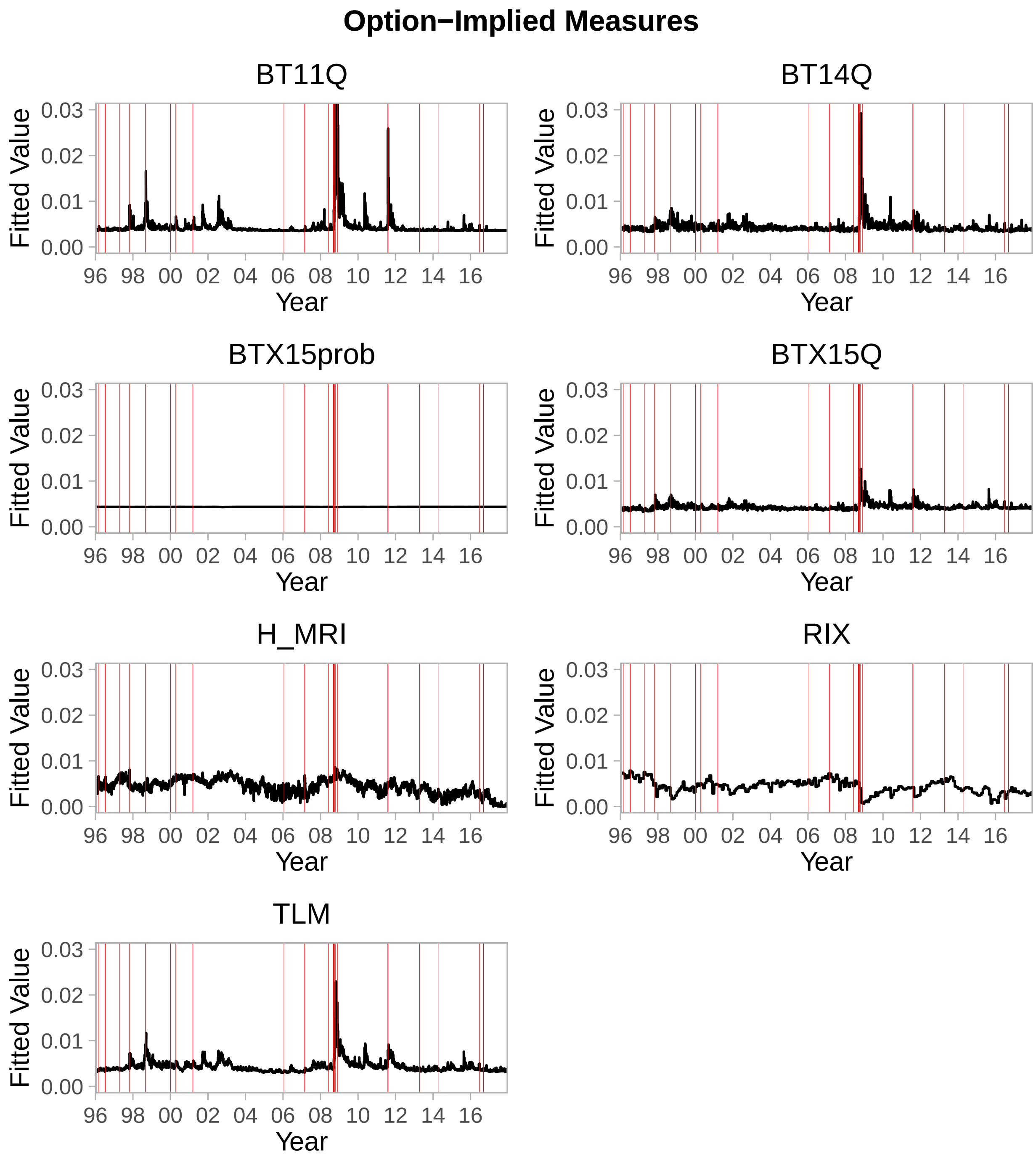

Figure A2. This figure displays the fitted values ($b \cdot TRM_t$) from the probit regression: $D_{t+\Delta t} = a + b \cdot TRM_t + \epsilon_{t+\Delta t}$, executed at the daily frequency. $D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. The conditional volatility is defined as the level of the VIX at the end of the previous day. TRM_t is the current observation of a tail risk measure. We indicate the actual crash realizations ($D_{t+\Delta t} = 1$) by vertical red lines. For $BT11Q$, the figure truncates values between October and December 2008. The largest peak occurs on October 13th, 2008, reaching 0.139.

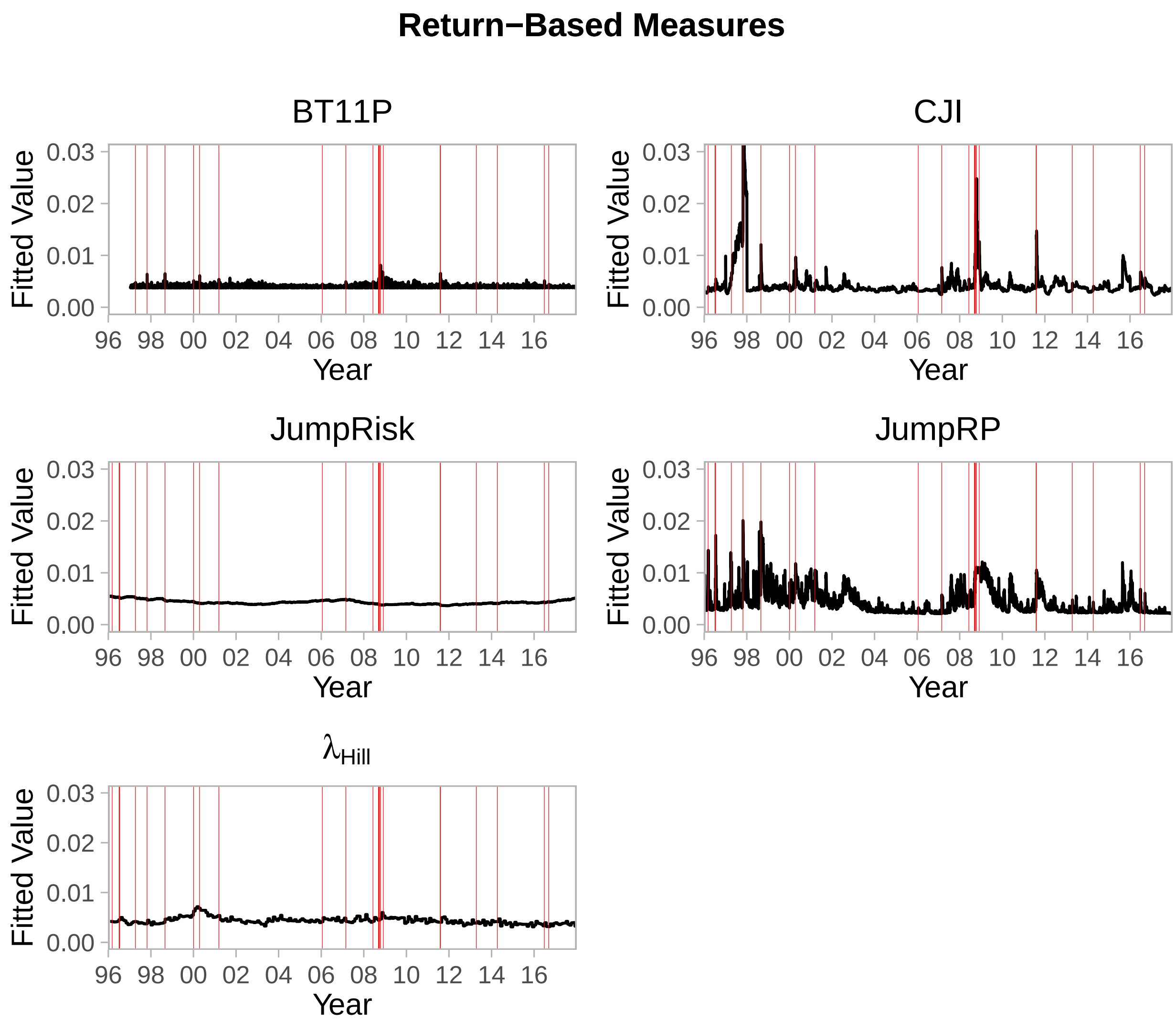

Figure A3. This figure displays the fitted values ($b \cdot TRM_t$) from the probit regression: $D_{t+\Delta t} = a + b \cdot TRM_t + \epsilon_{t+\Delta t}$, executed at the daily frequency. $D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. The conditional volatility is defined as the level of the VIX at the end of the previous day. TRM_t is the current observation of a tail risk measure. We indicate the actual crash realizations ($D_{t+\Delta t} = 1$) by vertical red lines. For CJI, the figure truncates values between October and November 1997. The largest peak occurs on October 30th, 1997, reaching 0.036.

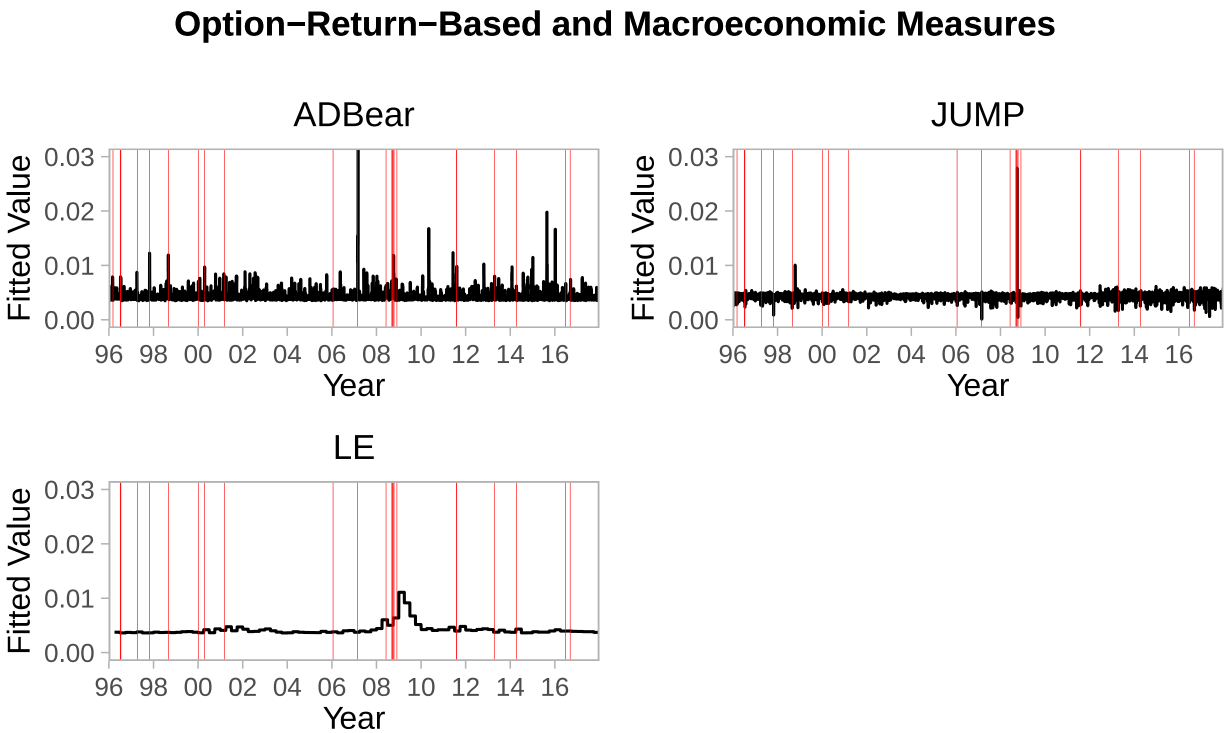

Figure A4. This figure displays the fitted values ($b \cdot TRM_t$) from the probit regression: $D_{t+\Delta t} = a + b \cdot TRM_t + \epsilon_{t+\Delta t}$, executed at the daily frequency. $D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. The conditional volatility is defined as the level of the VIX at the end of the previous day. TRM_t is the current observation of a tail risk measure. We indicate the actual crash realizations ($D_{t+\Delta t} = 1$) by vertical red lines.

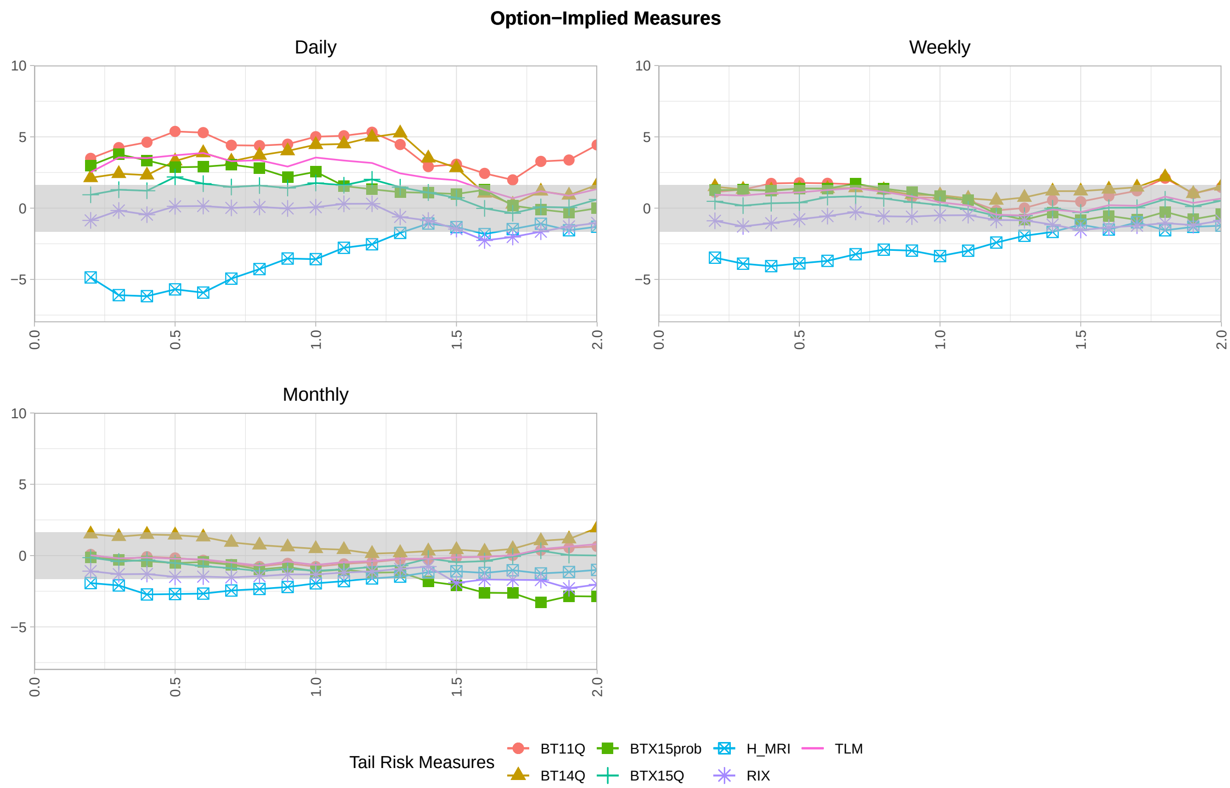

Figure A5. This figure presents the z-statistics from predictive probit regressions for different tail thresholds for option-implied tail risk measures. We perform the regression over different time intervals, from daily (top left) to annually (bottom). We perform single regressions of a dummy variable on each lagged tail risk measure. The dummy variable is 1 if the realized market excess return falls below the threshold defined by minus x times the current conditional volatility, with x shown on the horizontal axis ($x \in [0.2; 2]$). The conditional volatility is defined as the VIX at the end of the previous day. The gray shaded area denotes statistical insignificance at the 5% level. Different colors and point shapes indicate the different tail risk measures. The definitions of the tail risk measure acronyms are given in Table I.

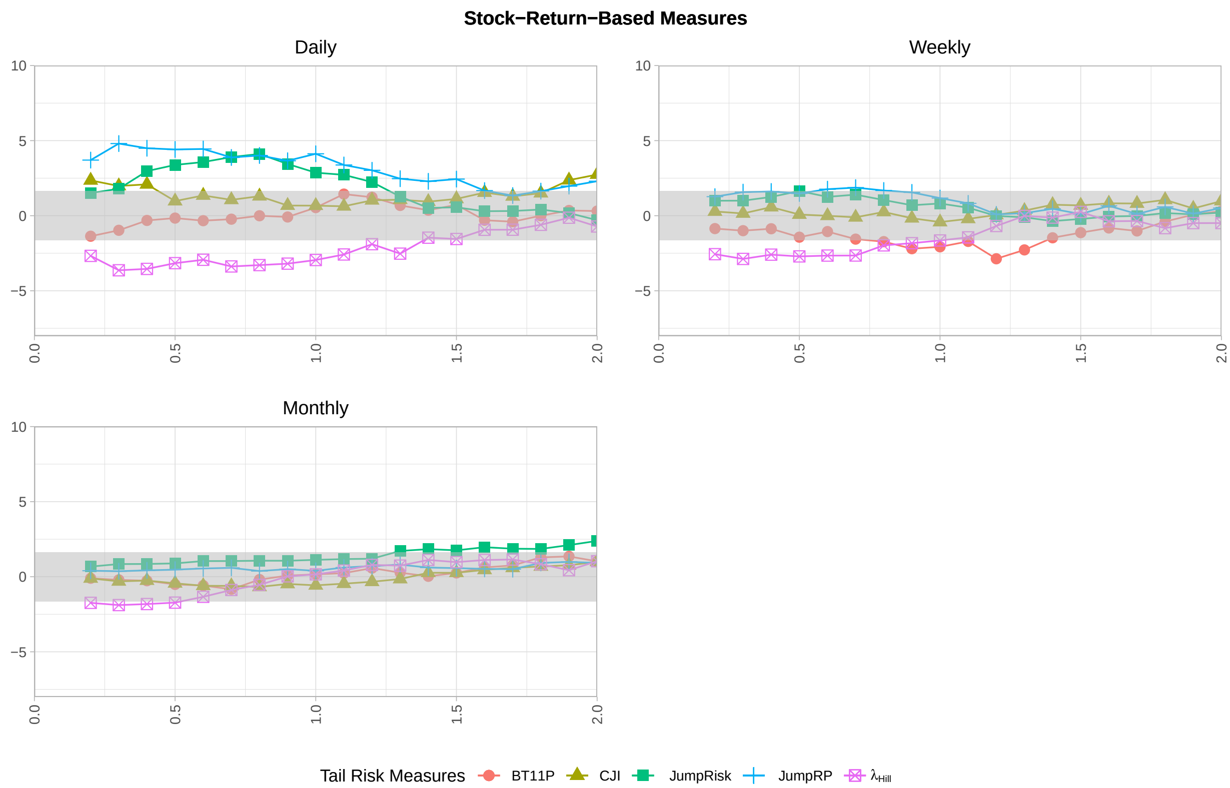

Figure A6. This figure presents the z-statistics from predictive probit regressions for different tail thresholds for return-based tail risk measures. We perform the regression over different time intervals, from daily (top left) to annually (bottom). We perform single regressions of a dummy variable on each lagged tail risk measure. The dummy variable is 1 if the realized market excess return falls below the threshold defined by minus x times the current conditional volatility, with x shown on the horizontal axis ($x \in [0.2; 2]$). The conditional volatility is defined as the VIX at the end of the previous day. The gray shaded area denotes statistical insignificance at the 5% level. Different colors and point shapes indicate the different tail risk measures. The definitions of the tail risk measure acronyms are given in Table I.

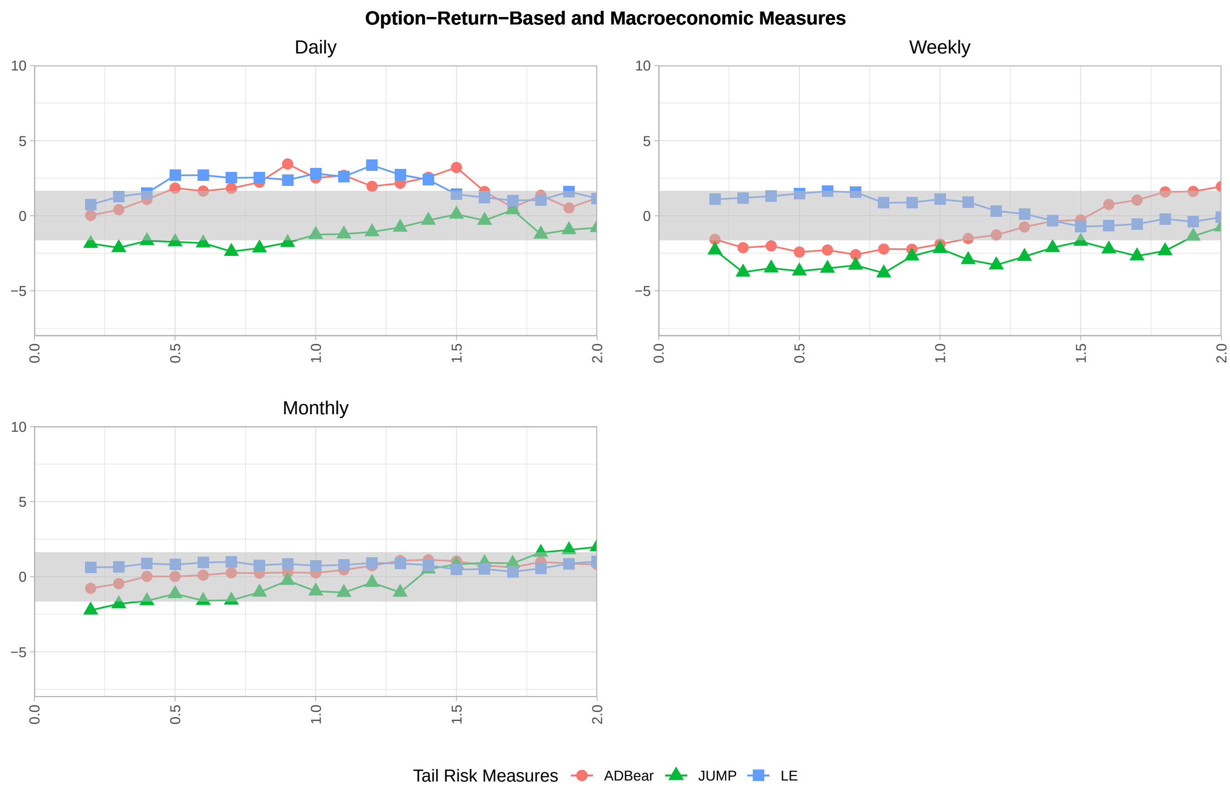

Figure A7. This figure presents the z-statistics from predictive probit regressions for different tail thresholds for option-return-based and macroeconomic tail risk measures. We perform the regression over different time intervals, from daily (top left) to annually (bottom). We perform single regressions of a dummy variable on each lagged tail risk measure. The dummy variable is 1 if the realized market excess return falls below the threshold defined by minus x times the current conditional volatility, with x shown on the horizontal axis ($x \in [0.2; 2]$). The conditional volatility is defined as the VIX at the end of the previous day. The gray shaded area denotes statistical insignificance at the 5% level. Different colors and point shapes indicate the different tail risk measures. The definitions of the tail risk measure acronyms are in Table I.

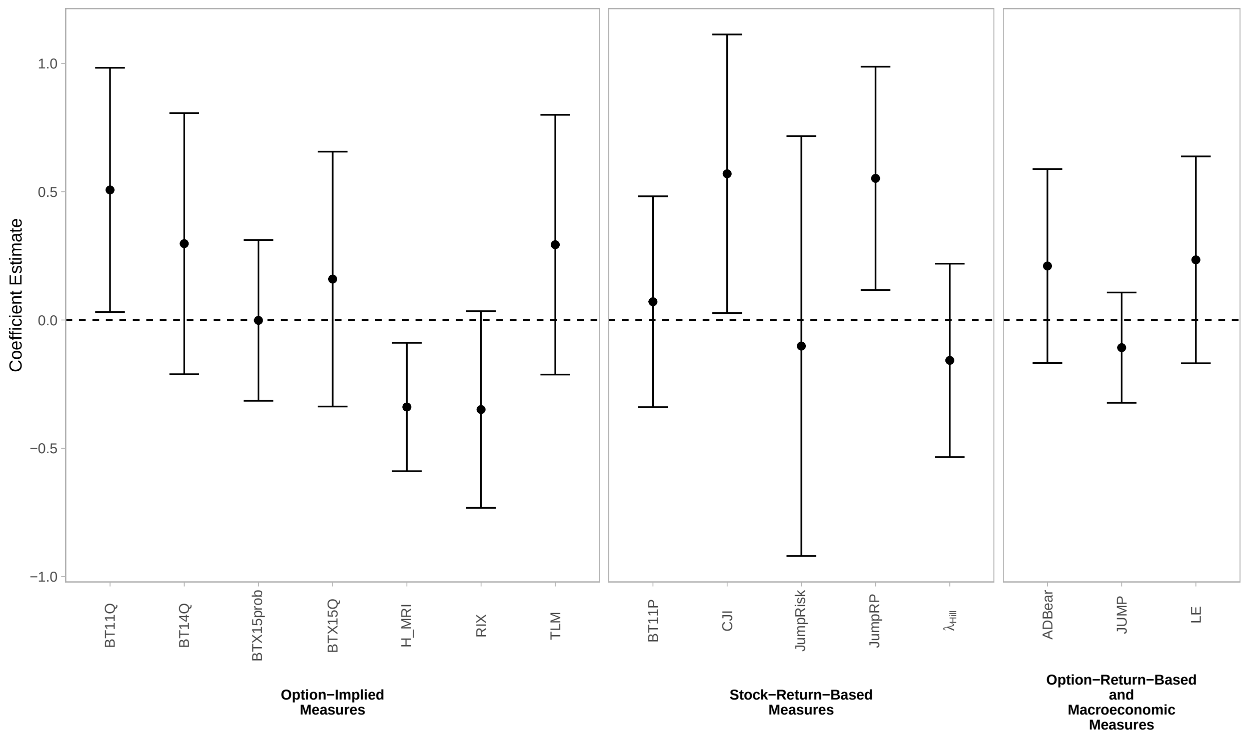

Figure A8. For the daily forecast horizon, this figure displays the coefficient estimate b from the regression: $TRM_t = a + b \cdot D_{t+\Delta t} + \epsilon_{t+\Delta t}$. TRM_t is the current observation of a tail risk measure. $D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. The conditional volatility is defined as the level of the VIX at the end of the previous day. We indicate the point estimate with a point. The 90% confidence interval around the point estimate is displayed by the vertical line. The confidence interval is based on Newey and West (1987) standard errors with 29 lags.

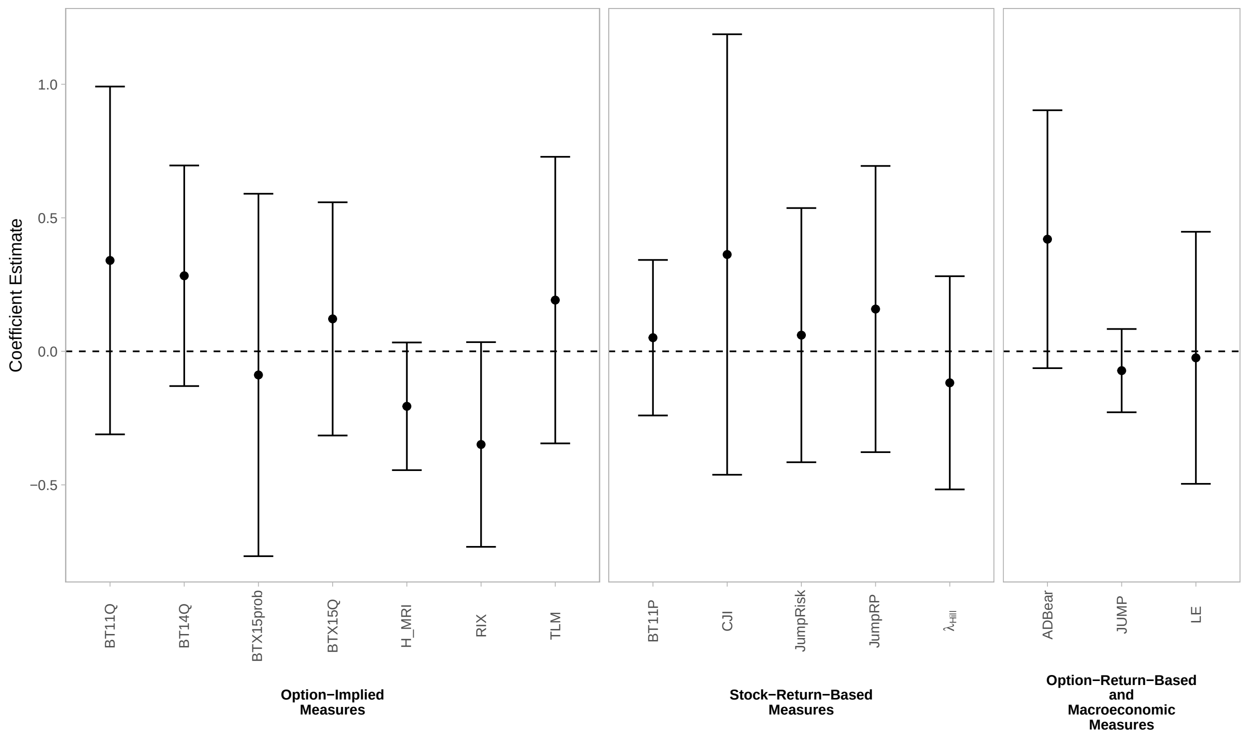

Figure A9. For the weekly forecast horizon, this figure displays the coefficient estimate b from the regression: $TRM_t = a + b \cdot D_{t+\Delta t} + \epsilon_{t+\Delta t}$. TRM_t is the current observation of a tail risk measure. $D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. The conditional volatility is defined as the level of the VIX at the end of the previous day. We indicate the point estimate with a point. The 90% confidence interval around the point estimate is displayed by the vertical line. The confidence interval is based on Newey and West (1987) standard errors with 29 lags.

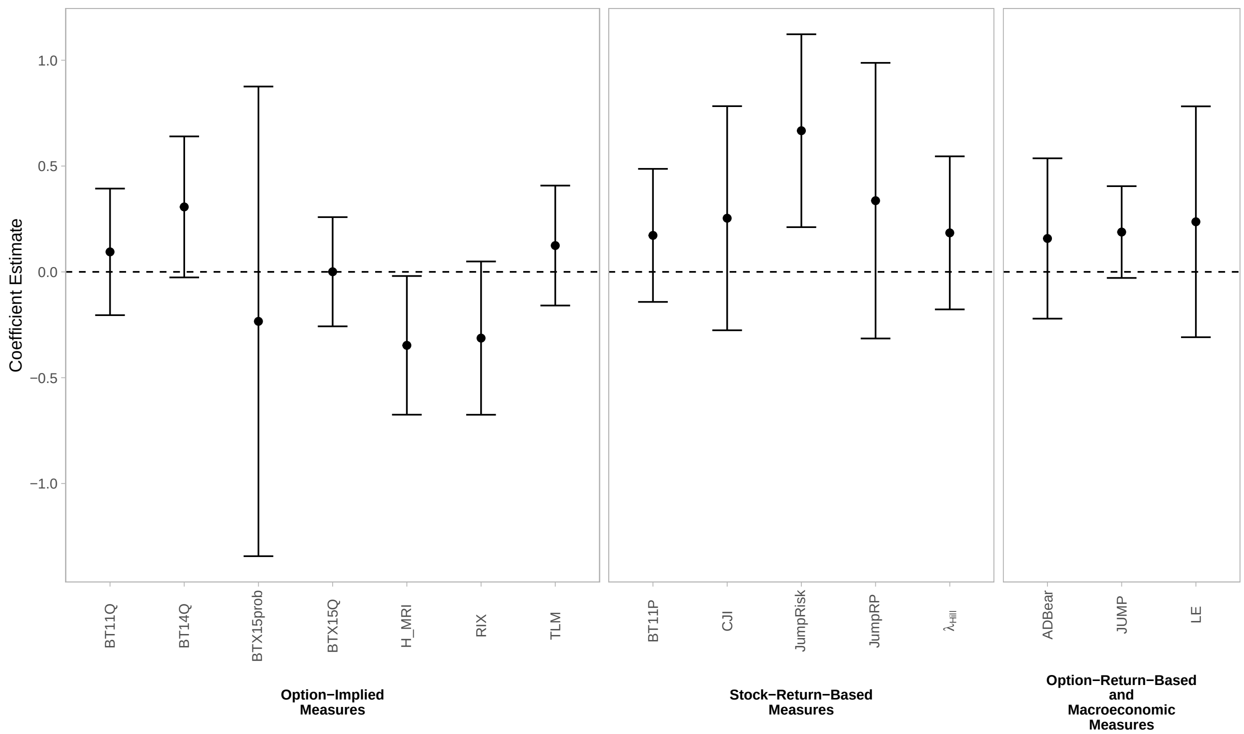

Figure A10. For the monthly forecast horizon, this figure displays the coefficient estimate b from the regression: $TRM_t = a + b \cdot D_{t+\Delta t} + \epsilon_{t+\Delta t}$. TRM_t is the current observation of a tail risk measure. $D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. The conditional volatility is defined as the level of the VIX at the end of the previous day. We indicate the point estimate with a point. The 90% confidence interval around the point estimate is displayed by the vertical line. The confidence interval is based on Newey and West (1987) standard errors with 29 lags.

Table A1 Cross-Sectional Return Predictability (Equally-Weighted)

This table presents the average annualized percentage excess returns of quintile portfolios sorted on the stock loadings on the different tail risk measures. Each month, we estimate the tail risk loadings (b^i) for each stock based on a rolling historical window:

$$R^i_{t+\Delta t} = a^i + b^i \cdot TRM_t + \epsilon^i_t,$$

$R^i_{t+\Delta t}$ is the excess return of stock i over the period between t and Δt. TRM_t is the current observation of a tail risk measure. We forecast stock returns at the daily frequency and use a window length of one month for all measures available at the daily frequency, and accordingly longer windows for measures available on lower frequencies. Based on their current b^i we then sort the stocks into quintile portfolios and obtain the equally-weighted portfolio excess return over the next month. We repeat the entire procedure in the next month. The $High - Low$ portfolio simultaneously buys the stocks in the portfolio with the highest b^i and sells those in the portfolio with the lowest b^i. In parentheses, we report robust Newey and West (1987) standard errors using 22 lags. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Low	(2)	(3)	(4)	$High$	$High - Low$
Group A - Option-Implied Measures						
$BT11Q$	10.97^{**}	10.43^{***}	9.76^{***}	8.59^{**}	5.46	-5.51^{***}
	(4.548)	(3.226)	(3.088)	(3.703)	(5.341)	(1.906)
$BT14Q$	10.97^{**}	9.30^{**}	9.04^{***}	9.36^{***}	5.45	-5.52^{**}
	(5.177)	(3.817)	(3.406)	(3.465)	(4.991)	(2.387)
$BTX15prob$	11.57^{**}	9.91^{***}	9.43^{***}	8.79^{**}	5.02	-6.56
	(5.019)	(3.352)	(3.207)	(3.813)	(5.492)	(4.247)
$BTX15Q$	6.82	8.94^{***}	9.33^{***}	10.14^{***}	9.97^{*}	3.15
	(4.837)	(3.226)	(3.235)	(3.613)	(5.354)	(2.774)
H_MRI	13.77^{**}	10.16^{**}	10.64^{**}	9.74^{**}	10.18	-3.59
	(5.989)	(4.324)	(4.157)	(4.720)	(6.201)	(2.816)
RIX	12.05^{**}	10.13^{***}	8.79^{**}	8.65^{**}	9.36	-2.68
	(4.740)	(3.596)	(3.493)	(4.162)	(6.210)	(2.811)
TLM	10.64^{**}	10.52^{***}	9.03^{***}	9.37^{**}	5.66	-4.98^{**}
	(4.587)	(3.244)	(3.295)	(3.801)	(5.222)	(2.407)
Group B - Stock-Return-Based Measures						
$BT11P$	9.50^{*}	10.51^{***}	8.71^{***}	8.31^{**}	6.79	-2.70
	(5.106)	(3.437)	(3.299)	(3.827)	(5.274)	(1.881)
CJI	7.82^{*}	9.28^{***}	9.63^{***}	9.93^{***}	8.56	0.74
	(4.422)	(3.318)	(3.196)	(3.617)	(5.333)	(1.908)
$JumpRisk$	11.18^{***}	10.11^{***}	9.60^{***}	8.72^{**}	5.59	-5.59^{**}
	(4.267)	(3.101)	(3.364)	(3.793)	(5.605)	(2.759)
$JumpRP$	10.67^{**}	9.63^{***}	9.24^{***}	8.97^{**}	6.70	-3.97^{*}
	(4.468)	(3.230)	(3.231)	(3.791)	(5.293)	(2.277)
λ_{Hill}	12.73^{**}	10.08^{**}	8.89^{**}	8.69^{**}	8.66	-4.07
	(5.984)	(4.021)	(3.597)	(3.665)	(5.305)	(3.460)
Group C - Option-Return-Based Measures						
$ADBear$	10.15^{**}	10.67^{***}	9.27^{***}	8.89^{**}	6.25	-3.90^{*}
	(4.884)	(3.234)	(3.219)	(3.732)	(5.006)	(2.108)
$JUMP$	10.15^{**}	10.03^{***}	8.93^{***}	8.82^{**}	7.28	-2.87
	(4.664)	(3.225)	(3.102)	(3.644)	(5.308)	(2.117)
Group D - Macroeconomic Measures						
LE	7.74	9.21^{**}	8.85^{***}	9.86^{***}	9.54^{**}	1.80
	(5.247)	(3.553)	(3.192)	(3.213)	(4.595)	(1.378)

Table A2 Cross-Sectional Return Predictability (Value-Weighted FF-5 Alphas)

This table presents the annualized percentage Fama and French (2015) 5-factor alphas of quintile portfolios sorted on the stock loadings on the different tail risk measures. Each month, we estimate the tail risk loadings (b^i) for each stock based on a rolling historical window:

$$R^i_{t+\Delta t} = a^i + b^i \cdot TRM_t + \epsilon^i_t,$$

$R^i_{t+\Delta t}$ is the excess return of stock i over the period between t and Δt. TRM_t is the current observation of a tail risk measure. We forecast stock returns at the daily frequency and use a window length of one month for all measures available at the daily frequency, and accordingly longer windows for measures available on lower frequencies. Based on their current b^i we then sort the stocks into quintile portfolios and obtain the value-weighted portfolio excess return over the next month. We repeat the entire procedure in the next month. The $High - Low$ portfolio simultaneously buys the stocks in the portfolio with the highest b^i and sells those in the portfolio with the lowest b^i. In parentheses, we report robust Newey and West (1987) standard errors using 22 lags. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Low	(2)	(3)	(4)	*High*	*High − Low*
Group A - Option-Implied Measures						
$BT11Q$	6.59^{**}	3.49^{**}	1.85	−0.32	−3.32	-9.91^{***}
	(2.931)	(1.639)	(1.285)	(1.365)	(2.470)	(2.049)
$BT14Q$	5.69^{*}	2.02	1.21	1.76	−2.77	-8.46^{***}
	(3.416)	(1.983)	(1.621)	(1.475)	(2.416)	(3.129)
$BTX15prob$	6.61^{*}	2.94^{*}	1.57	0.31	−3.26	-9.87^{**}
	(3.757)	(1.685)	(1.510)	(1.714)	(2.995)	(4.875)
$BTX15Q$	0.86	1.65	1.21	1.81	2.76	1.90
	(2.456)	(1.701)	(1.493)	(1.513)	(3.162)	(3.056)
H_MRI	4.94	1.89	2.44^{**}	1.77	0.52	−4.42
	(3.268)	(1.299)	(1.097)	(1.651)	(2.258)	(2.759)
RIX	3.40	2.20	1.77	1.70	3.80	0.40
	(2.468)	(1.638)	(1.644)	(1.847)	(3.242)	(1.912)
TLM	6.17^{**}	3.43^{**}	0.87	0.63	−2.81	-8.97^{***}
	(3.011)	(1.700)	(1.404)	(1.370)	(2.501)	(2.428)
Group B - Stock-Return-Based Measures						
$BT11P$	4.34	3.61^{*}	1.21	0.17	−0.83	-5.18^{***}
	(2.903)	(1.887)	(1.475)	(1.521)	(2.317)	(1.991)
CJI	2.31	1.90	1.81	1.50	0.77	−1.53
	(2.659)	(1.524)	(1.485)	(1.435)	(2.731)	(2.390)
$JumpRisk$	6.06^{*}	3.14^{*}	1.66	0.01	−2.58	-8.64^{***}
	(3.162)	(1.784)	(1.613)	(1.283)	(2.137)	(2.572)
$JumpRP$	5.89^{*}	2.29	1.42	0.28	−1.59	-7.48^{***}
	(3.149)	(1.682)	(1.429)	(1.324)	(2.172)	(2.300)
λ_{Hill}	6.84^{**}	2.97^{*}	1.80	0.88	0.55	-6.29^{**}
	(3.462)	(1.769)	(1.712)	(1.527)	(2.943)	(2.957)
Group C - Option-Return-Based Measures						
$ADBear$	5.35^{*}	3.67^{*}	1.21	0.17	−2.09	-7.44^{***}
	(2.972)	(1.909)	(1.401)	(1.291)	(2.235)	(2.078)
$JUMP$	3.81	2.00	0.80	0.97	0.71	−3.10
	(2.902)	(1.522)	(1.364)	(1.388)	(2.377)	(1.926)
Group D - Macroeconomic Measures						
LE	1.82	1.28	0.81	1.92	2.45	0.63
	(2.533)	(1.577)	(1.440)	(1.355)	(2.531)	(1.275)

Table A3 Return Predictability: Pre−2008

This table presents the coefficients from a return predictability regression for the period from 1996 to 2007. We perform single regressions of the market excess returns on each lagged tail risk measure:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot Controls_t + \epsilon_{t+\Delta t}.$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is the current observation of a tail risk measure. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), (iii) one-month (*Monthly*), and (iv) one-year (*Annually*). In parentheses, we present robust Newey and West (1987) standard errors with lag length chosen to be the maximum of 29 and the number of overlapping observations. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. "*PCOneAll*", "*PCOneOption*", "*PCOneStReturn*", and "*PCOneOpReturn*" denote the first PCs of all measures, option-implied, stock-return-based, and option-return-based tail risk measures, respectively. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2	*Annually*	R^2
Group A - Option-Implied Measures								
BT11Q	18.07**	0.24	18.56**	1.27	10.19**	2.68	−0.63	0.38
	(8.136)		(7.838)		(4.758)		(2.199)	
BT14Q	−3.07	0.01	−1.45	0.03	−1.73	0.19	−1.92*	0.69
	(7.754)		(6.436)		(3.356)		(1.208)	
BTX15prob	11.01	0.09	10.82*	0.44	9.66**	1.95	−0.59	0.43
	(8.350)		(7.470)		(5.408)		(3.645)	
BTX15Q	1.83	0.03	−5.69	0.07	−6.39*	0.43	−3.41**	1.89
	(7.282)		(6.888)		(3.740)		(1.605)	
H_MRI	−5.08	0.02	−1.94	0.02	0.01	0.06	5.63**	4.42
	(4.913)		(4.493)		(3.623)		(2.845)	
RIX	−0.89	0.02	−2.63	0.08	−8.12*	0.52	−6.36**	4.34
	(6.144)		(6.275)		(4.829)		(2.818)	
TLM	16.21*	0.16	13.60*	0.65	2.07	0.97	−5.30*	2.48
	(8.962)		(9.443)		(5.497)		(3.300)	
Group B - Stock-Return-Based Measures								
BT11P	15.60***	0.28	12.93***	1.03	4.46***	0.57	0.08	0.01
	(6.366)		(2.417)		(1.265)		(0.534)	
CJI	5.94	0.05	6.52*	0.31	2.82	0.52	−0.63	0.14
	(4.676)		(4.573)		(3.542)		(1.490)	
JumpRisk	−7.81	0.03	−8.78*	0.16	−12.32***	1.48	−15.63***	27.33
	(6.282)		(5.969)		(5.014)		(2.055)	
JumpRP	16.27**	0.18	13.67**	0.72	3.92	0.89	−4.41*	2.47
	(6.744)		(5.615)		(4.538)		(2.579)	
λ_{Hill}	3.61	0.01	6.54	0.17	7.57**	0.92	9.41***	21.83
	(5.214)		(5.115)		(4.287)		(1.809)	
Group C - Option-Return-Based Measures								
ADBear	18.45***	0.43	15.47***	1.59	4.71***	0.72	−0.04	0.01
	(5.148)		(3.943)		(1.850)		(0.452)	
JUMP	9.83**	0.12	4.89**	0.15	2.37***	0.17	0.41**	0.03
	(6.150)		(2.508)		(0.879)		(0.236)	
Group D - Macroeconomic Measures								
LE	−2.62	0.01	−2.43	0.06	−4.48	0.73	−8.55***	18.35
	(5.192)		(5.234)		(4.090)		(1.721)	
PCOneAll	24.07***	0.27	19.57**	1.01	7.84	1.73	−5.47	2.49
	(9.069)		(9.206)		(6.056)		(3.558)	
PCOneOption	12.92	0.12	9.23	0.40	1.08	0.92	−6.30*	2.89
	(10.030)		(10.138)		(6.024)		(3.506)	
PCOneStReturn	15.66***	0.20	12.11**	0.67	2.88	0.36	−6.06***	8.54
	(5.916)		(4.921)		(3.733)		(1.961)	
PCOneOpReturn	17.83***	0.40	12.85***	1.09	4.47***	0.64	0.23	0.01
	(5.993)		(3.718)		(1.565)		(0.320)	
Controls	*Yes*		*Yes*		*Yes*		*Yes*	

Table A4 Multiple Return Predictability: Pre−2008

This table presents the coefficients from a return predictability regression for the period from 1996 to 2007. We perform multiple regressions of the market excess returns on lagged tail risk measures:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot Controls_t + \epsilon_{t+\Delta t}.$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is a vector of the current observations of the tail risk measures. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), (iii) one-month (*Monthly*), and (iv) one-year (*Annually*). For each forecast horizon, we first perform variable selection based on the PcGets algorithm. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with lag length chosen to be the maximum of 29 and the number of overlapping observations. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2	*Annually*	R^2
Group A - Option-Implied Measures								
$BT11Q$			17.74^{**}	1.44	18.01^{***}	2.44	6.01^{***}	4.41
			(6.789)		(6.427)		(1.122)	
$BT14Q$					−1.53	0.13		
					(2.384)			
$BTX15prob$					13.42^{**}	1.72		
					(6.468)			
$BTX15Q$					-5.18^{*}	0.51	-2.49^{**}	0.62
					(3.427)		(1.103)	
H_MRI					−4.44	0.32		
					(3.598)			
RIX					−0.22	0.17		
					(4.655)			
TLM	12.33^{**}	0.15			-12.69^{*}	0.79		
	(5.096)				(8.570)			
Group B - Stock-Return-Based Measures								
$BT11P$					1.42	0.17		
					(1.202)			
CJI					3.43	0.32		
					(4.034)			
$JumpRisk$					−9.03	0.94	-5.98^{**}	9.05
					(9.147)		(2.629)	
$JumpRP$					3.86	0.48		
					(8.952)			
λ_{Hill}					5.00	0.62	6.30^{***}	16.36
					(6.394)		(1.691)	
Group C - Option-Return-Based Measures								
$ADBear$			11.54^{***}	1.26	1.28	0.20		
			(4.109)		(1.880)			
$JUMP$					−0.52	0.03		
					(0.923)			
Group D - Macroeconomic Measures								
LE					2.53	0.45	−3.19	9.80
					(4.585)		(1.792)	
Controls	*Yes*		*Yes*		*Yes*		*Yes*	

Table A5 Return Predictability: Post−2008

This table presents the coefficients from a return predictability regression for the period from 2008 to 2017. We perform single regressions of the market excess returns on each lagged tail risk measure:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot Controls_t + \epsilon_{t+\Delta t}.$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is the current observation of a tail risk measure. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), (iii) one-month (*Monthly*), and (iv) one-year (*Annually*). In parentheses, we present robust Newey and West (1987) standard errors with lag length chosen to be the maximum of 29 and the number of overlapping observations. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. "*PCOneAll*", "*PCOneOption*", "*PCOneStReturn*", and "*PCOneOpReturn*" denote the first PCs of all measures, option-implied, stock-return-based, and option-return-based tail risk measures, respectively. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2	*Annually*	R^2
Group A - Option-Implied Measures								
BT11Q	64.68***	0.98	24.49**	0.64	8.83	0.39	4.96***	6.38
	(14.971)		(9.276)		(6.878)		(1.314)	
BT14Q	15.72*	0.09	6.35	0.07	−5.54*	1.10	3.07**	3.95
	(10.345)		(6.456)		(3.870)		(1.089)	
BTX15prob	26.53***	0.13	17.20**	0.34	15.33**	1.34	1.31	1.55
	(11.021)		(9.209)		(7.874)		(1.882)	
BTX15Q	2.57	0.00	2.17	0.03	−0.41	0.14	3.13**	7.88
	(11.498)		(9.171)		(4.422)		(1.491)	
H_MRI	−7.85	0.03	−2.57	0.05	0.63	0.10	4.65**	2.06
	(5.962)		(4.966)		(4.494)		(1.457)	
RIX	8.22	0.02	13.18*	0.30	13.46**	1.71	0.00	5.01
	(9.907)		(8.315)		(5.806)		(1.712)	
TLM	55.75***	0.56	29.54***	0.87	11.09*	0.57	4.08**	5.00
	(18.324)		(11.359)		(6.902)		(2.013)	
Group B - Stock-Return-Based Measures								
BT11P	37.03***	1.07	15.49***	1.00	3.57*	0.17	1.09**	0.56
	(8.989)		(5.357)		(2.181)		(0.366)	
CJI	−0.03	0.00	−6.90	0.27	−7.50	1.38	0.49	1.69
	(10.334)		(9.809)		(5.857)		(1.227)	
JumpRisk	11.23	0.03	5.77	0.13	−3.97	0.34	−4.25	3.73
	(13.992)		(15.032)		(13.690)		(3.347)	
JumpRP	26.29***	0.12	18.29**	0.37	9.84*	0.50	1.24	1.81
	(8.508)		(7.541)		(6.172)		(1.492)	
λ_{Hill}	−14.14**	0.03	−16.11***	0.24	−21.01***	2.51	−3.51**	1.38
	(6.849)		(6.142)		(5.378)		(1.403)	
Group C - Option-Return-Based Measures								
ADBear	20.27**	0.37	11.19**	0.67	2.12	0.09	0.64	0.10
	(8.588)		(4.838)		(2.349)		(0.479)	
JUMP	−3.56	0.01	6.58***	0.24	0.03	0.00	0.04	0.00
	(9.473)		(1.944)		(1.017)		(0.095)	
Group D - Macroeconomic Measures								
LE	7.19	0.01	4.64	0.07	13.95	0.72	2.52	2.93
	(15.163)		(15.484)		(10.704)		(2.627)	
PCOneAll	−62.98***	0.42	−32.17***	0.62	−13.49*	0.57	−4.72**	4.91
	(18.664)		(11.067)		(8.141)		(2.107)	
PCOneOption	48.25**	0.36	24.29**	0.49	8.80	0.35	4.23*	6.04
	(18.070)		(10.266)		(7.642)		(2.089)	
PCOneStReturn	−42.56***	0.26	−20.61*	0.35	−9.37	0.45	−2.30	2.37
	(11.598)		(12.301)		(8.930)		(1.782)	
PCOneOpReturn	10.78	0.10	11.48***	0.71	1.39	0.03	0.44	0.05
	(9.165)		(3.575)		(1.909)		(0.375)	
Controls	*Yes*		*Yes*		*Yes*		*Yes*	

Table A6 Multiple Return Predictability: Post−2008

This table presents the coefficients from a return predictability regression for the period from 2008 to 2017. We perform multiple regressions of the market excess returns on lagged tail risk measures:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot Controls_t + \epsilon_{t+\Delta t}.$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is a vector of the current observations of the tail risk measures. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), (iii) one-month (*Monthly*), and (iv) one-year (*Annually*). For each forecast horizon, we first perform variable selection based on the PcGets algorithm. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with lag length chosen to be the maximum of 29 and the number of overlapping observations. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2	*Annually*	R^2
Group A - Option-Implied Measures								
$BT11Q$	53.43^{*}	0.59					3.43^{**}	3.79
	(26.057)						(1.322)	
$BT14Q$							0.83	2.00
							(0.705)	
$BTX15prob$								
$BTX15Q$	-83.25^{***}	0.68	-36.89^{***}	0.70				
	(22.970)		(12.843)					
H_MRI							4.39^{***}	2.90
							(1.092)	
RIX								
TLM	79.31^{*}	0.43	46.30^{***}	0.95				
	(40.381)		(14.624)					
Group B - Stock-Return-Based Measures								
$BT11P$	22.67^{**}	0.68					0.38	0.26
	(8.962)						(0.283)	
CJI								
$JumpRisk$								
$JumpRP$								
λ_{Hill}								
Group C - Option-Return-Based Measures								
$ADBear$								
$JUMP$			6.22^{***}	0.23				
			(1.956)					
Group D - Macroeconomic Measures								
LE								
Controls	*Yes*		*Yes*		*Yes*		*Yes*	

Table A7 Multiple Prediction of Tail Events: Jackknife Procedure

This table presents the coefficients from the predictive probit regressions. We perform multiple probit regressions of a dummy variable on lagged tail risk measures:

$$D_{t+\Delta t} = a + b \cdot TRM_t + \epsilon_{t+\Delta t}.$$

$D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. The conditional volatility is defined as the level of the VIX at the previous day. TRM_t is a vector of the current observations of the tail risk measures. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), and (iii) one-month (*Monthly*). For each forecast horizon, we first perform variable selection based on a jackknife procedure. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the partial McFadden R^2s, obtained by dominance analysis, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2
Group A - Option-Implied Measures						
BT11Q						
BT14Q	0.13^{**}	1.87			0.12	0.73
	(0.050)				(0.075)	
BTX15prob	−0.23	0.74	−0.13	0.39	-0.71^{**}	3.47
	(0.147)		(0.129)		(0.279)	
BTX15Q						
H_MRI	−0.10	0.70	−0.11	0.37		
	(0.134)		(0.070)			
RIX	−0.16	0.98	−0.08	0.62	-0.37^{**}	2.57
	(0.151)		(0.139)		(0.154)	
TLM						
Group B - Stock-Return-Based Measures						
BT11P			0.09^{**}	0.82		
			(0.044)			
CJI			0.08	0.49		
			(0.092)			
JumpRisk					0.51^{**}	5.88
					(0.205)	
JumpRP	0.15	1.42			0.44^{*}	2.54
	(0.151)				(0.254)	
λ_{Hill}					0.21^{***}	0.96
					(0.063)	
Group C - Option-Return-Based Measures						
ADBear	0.03	1.02	0.06	0.56		
	(0.084)		(0.067)			
JUMP	0.10^{**}	3.45	-0.13^{***}	1.26	0.00	0.22
	(0.042)		(0.041)		(0.033)	
Group D - Macroeconomic Measures						
LE	0.08	0.43	−0.04	0.09		
	(0.101)		(0.109)			

Table A8 Multiple Predictability of Left Tail Variation: Jackknife Procedure

This table presents the coefficients from a predictive regression for future left tail variation. We perform multiple regressions of the realized left tail variation on the lagged tail risk measures:

$$LTV^{\mathbb{P}}_{t+\Delta t} = a + b \cdot TRM_t + c \cdot LTV^{\mathbb{P}}_t + d \cdot VIX_t + \epsilon_{t+\Delta t}.$$

TRM_t is a vector of the current observations of the tail risk measures. We control for the lagged left tail variation $LTV^{\mathbb{P}}_t$ and the current level of the VIX (VIX_t). We use three different forecast horizons Δt: (i) one-day ($Daily$), (ii) one-week ($Weekly$), and (iii) one-month ($Monthly$). For each forecast horizon, we first perform variable selection based on a jackknife procedure. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2
Group A - Option-Implied Measures						
$BT11Q$	0.13^{**}	1.78	0.20^{**}	5.16	0.15^{**}	6.89
	(0.051)		(0.073)		(0.064)	
$BT14Q$	0.06^{***}	1.28	0.16^{**}	4.17	0.10^{**}	4.07
	(0.028)		(0.086)		(0.054)	
$BTX15prob$	-0.11^{*}	0.48	-0.07	0.91	-0.10^{*}	1.34
	(0.069)		(0.058)		(0.069)	
$BTX15Q$			0.06	2.47		
			(0.067)			
H_MRI						
RIX	-0.08^{**}	0.30	-0.12^{**}	0.70	-0.12^{**}	0.81
	(0.038)		(0.059)		(0.054)	
TLM	0.14^{*}	1.36				
	(0.090)					
Group B - Stock-Return-Based Measures						
$BT11P$	-0.05	0.13	0.04^{**}	0.83	0.06^{**}	1.34
	(0.034)		(0.024)		(0.028)	
CJI	0.03	0.36	0.07	1.21	0.08^{*}	1.82
	(0.029)		(0.051)		(0.055)	
$JumpRisk$	0.05^{***}	0.36	0.09^{***}	1.30	0.11^{**}	2.65
	(0.021)		(0.040)		(0.053)	
$JumpRP$						
λ_{Hill}						
Group C - Option-Return-Based Measures						
$ADBear$						
$JUMP$	-0.01	0.01				
	(0.010)					
Group D - Macroeconomic Measures						
LE	0.03	0.41	0.07^{**}	1.58	0.16^{***}	4.41
	(0.024)		(0.038)		(0.057)	
Controls	*Yes*		*Yes*		*Yes*	

Table A9 Multiple Return Predictability: Jackknife Procedure

This table presents the coefficients from a return predictability regression. We perform multiple regressions of the market excess returns on lagged tail risk measures:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot Controls_t + \epsilon_{t+\Delta t}.$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is a vector of the current observations of the tail risk measures. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), (iii) one-month (*Monthly*), and (iv) one-year (*Annually*). For each forecast horizon, we first perform variable selection based on a jackknife procedure. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with lag length chosen to be the maximum of 29 and the number of overlapping observations. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2	*Annually*	R^2
Group A - Option-Implied Measures								
$BT11Q$	36.50^{***}	0.25					4.83^{**}	4.14
	(9.343)						(1.631)	
$BT14Q$	-9.15	0.04			-3.03	0.36		
	(7.750)				(2.532)			
$BTX15prob$					13.49^{***}	1.89		
					(4.676)			
$BTX15Q$	-21.46^{***}	0.10	-8.60^{*}	0.14	-4.06^{*}	0.20	0.40	2.98
	(7.587)		(5.801)		(2.475)		(2.096)	
H_MRI								
RIX			5.35	0.22	1.04	0.75		
			(5.495)		(3.808)			
TLM								
Group B - Stock-Return-Based Measures								
$BT11P$	10.81	0.14	5.82^{*}	0.22				
	(9.500)		(3.292)					
CJI								
$JumpRisk$					-2.13	0.19	-6.13^{**}	2.92
					(4.402)		(3.024)	
$JumpRP$								
λ_{Hill}							2.21^{*}	7.70
							(1.519)	
Group C - Option-Return-Based Measures								
$ADBear$	6.68	0.09	6.63^{**}	0.29				
	(6.313)		(3.101)					
$JUMP$	-13.78^{**}	0.14	2.68	0.10	1.53^{***}	0.05		
	(6.561)		(3.067)		(0.639)			
Group D - Macroeconomic Measures								
LE								
Controls	*Yes*		*Yes*		*Yes*		*Yes*	

Table A10 Multiple Tail Return Predictability: Jackknife Procedure

This table presents the coefficients from a return predictability regression. We perform multiple regressions of the market excess returns on lagged tail risk measures, while separating crash and non-crash periods:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot TRM_t \cdot D_{t+\Delta t} + d \cdot D_{t+\Delta t} + e \cdot Controls_t + \epsilon_{t+\Delta t},$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is a vector of the current observations of the tail risk measures. $D_{t+\Delta t}$ is 1 if the realized market excess return falls below the threshold defined by minus two times the current conditional volatility. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use three different forecast horizons Δt: (i) one-day ($Daily$), (ii) one-week ($Weekly$), and (iii) one-month ($Monthly$). For each forecast horizon, we first perform variable selection based on the jackknife algorithm. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	c	$R^2(b)$	$R^2(c)$	Weekly	c	$R^2(b)$	$R^2(c)$	Monthly	c	$R^2(b)$	$R^2(c)$
Group A - Option-Implied Measures												
$BT11Q$	32.47***	73	0.20	0.30		−119***		1.61				
	(9.417)	(66.251)				(15.350)						
$BT14Q$	-9.43	−123***	0.05	0.14					-2.65		0.31	
	(7.893)	(47.453)							(2.473)			
$BTX15prob$									11.57***		1.65	
									(4.292)			
$BTX15Q$	−22.85***		0.11		-6.96		0.08		−4.68**		0.18	
	(7.753)				(4.812)				(2.535)			
H_MRI												
RIX	8.05		0.06									
	(7.275)											
TLM												
Group B - Stock-Return-Based Measures												
$BT11P$	12.15		0.15		5.17*		0.21					
	(9.684)				(3.278)							
CJI		191***		0.51								
		(38.943)										
$JumpRisk$										−61***		4.01
										(22.157)		
$JumpRP$		211***		0.55								
		(62.152)										
λ_{Hill}												
Group C - Option-Return-Based Measures												
$ADBear$	6.53		0.09		8.39***		0.37		3.41**		0.21	
	(6.551)				(2.866)				(1.506)			
$JUMP$	−14.13**		0.14		2.10	18	0.09	0.05	1.16*		0.04	
	(6.531)				(2.836)	(41.763)			(0.689)			
Group D - Macroeconomic Measures												
LE												
Controls	Yes				Yes				Yes			

Table A11 Prediction of Tail Events (Number of Jumps)

This table presents the coefficients from a predictive regression for the number of future negative jumps. We perform single regressions of the realized number of negative jumps (NLJ) on each lagged tail risk measure:

$$NLJ_{t+\Delta t} = a + b \cdot TRM_t + c \cdot NLJ_t + d \cdot VIX_t + \epsilon_{t+\Delta t}.$$

TRM_t is the current observation of a tail risk measure. We control for the lagged number of negative jumps NLJ_t and the current level of the VIX (VIX_t). We use three different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), and (iii) one-month (*Monthly*). In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2
Group A - Option-Implied Measures						
BT11Q	0.16^{***}	1.39	0.78^{***}	3.63	1.69^{***}	5.41
	(0.040)		(0.173)		(0.700)	
BT14Q	0.07^{***}	0.56	0.27^{***}	1.51	0.08	2.69
	(0.025)		(0.106)		(0.294)	
BTX15prob	−0.01	1.48	−0.12	4.34	0.08	7.05
	(0.040)		(0.169)		(0.618)	
BTX15Q	0.05^{**}	0.76	0.27^{**}	1.88	0.14	3.28
	(0.029)		(0.115)		(0.405)	
H_MRI	0.00	0.48	0.00	1.35	0.02	2.27
	(0.023)		(0.091)		(0.349)	
RIX	−0.03	0.60	−0.12	1.66	−0.13	2.54
	(0.030)		(0.121)		(0.487)	
TLM	−0.04	2.04	−0.06	5.38	−1.20	8.50
	(0.078)		(0.320)		(1.253)	
Group A - Option-Implied Measures						
BT11P	-0.10^{***}	1.16	-0.28^{***}	1.92	-0.57^{***}	1.58
	(0.018)		(0.053)		(0.126)	
CJI	0.04^{**}	0.25	0.20^{**}	0.60	0.38	0.93
	(0.025)		(0.096)		(0.378)	
JumpRisk	-0.11^{***}	1.52	-0.38^{***}	3.71	-0.96^{**}	5.56
	(0.027)		(0.110)		(0.484)	
JumpRP	-0.23^{***}	3.14	-0.72^{***}	7.20	-1.81^{***}	9.91
	(0.043)		(0.198)		(0.677)	
λ_{Hill}	0.03	0.46	0.10	1.20	0.39	2.39
	(0.031)		(0.128)		(0.482)	
Group A - Option-Implied Measures						
ADBear	-0.10^{***}	1.09	-0.42^{***}	2.37	-1.10^{***}	1.72
	(0.017)		(0.062)		(0.186)	
JUMP	-0.04^{***}	0.20	-0.15^{***}	0.30	-0.16^{*}	0.07
	(0.016)		(0.044)		(0.095)	
Group A - Option-Implied Measures						
LE	0.02	0.47	0.09	1.25	0.59^{*}	1.95
	(0.024)		(0.095)		(0.382)	
PCOneAll	-0.19^{**}	2.34	-0.50^{*}	5.94	−1.61	9.12
	(0.091)		(0.355)		(1.296)	
PCOneOption	0.14^{**}	1.70	0.61^{**}	4.52	0.55	7.32
	(0.071)		(0.285)		(1.060)	
PCOneStReturn	-0.26^{***}	3.50	-0.83^{***}	7.88	-2.24^{***}	10.97
	(0.040)		(0.154)		(0.519)	
PCOneOpReturn	-0.09^{***}	0.90	-0.36^{***}	1.82	-0.80^{***}	1.01
	(0.017)		(0.054)		(0.166)	
Controls	*Yes*		*Yes*		*Yes*	

Table A12 Predictability of Left Tail Variation (Including Overnight Returns)

This table presents the coefficients from a predictive regression for future left tail variation, including the overnight variation. We perform single regressions of the standardized realized left tail variation on each lagged tail risk measure:

$$LTV^{\mathbb{P}}_{t+\Delta t} = a + b \cdot TRM_t + c \cdot LTV^{\mathbb{P}}_t + d \cdot VIX_t + \epsilon_{t+\Delta t}.$$

TRM_t is the current observation of a tail risk measure. We control for the lagged left tail variation $LTV^{\mathbb{P}}_t$ and the current level of the VIX (VIX_t). We use three different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), and (iii) one-month (*Monthly*). In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the wild bootstrap of Rapach et al. (2013). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. "*PCOneAll*", "*PCOneOption*", "*PCOneStReturn*", and "*PCOneOpReturn*" denote the first PCs of all measures, option-implied, stock-return-based, and option-return-based tail risk measures, respectively. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2
Group A - Option-Implied Measures						
BT11Q	0.44***	9.62	0.41***	17.31	0.26*	15.18
	(0.120)		(0.098)		(0.152)	
BT14Q	0.13**	4.86	0.13**	9.67	0.04	7.34
	(0.071)		(0.060)		(0.036)	
BTX15prob	−0.19**	2.80	−0.20*	5.43	−0.24**	5.62
	(0.099)		(0.099)		(0.138)	
BTX15Q	0.12**	4.78	0.11**	9.36	0.03	7.74
	(0.053)		(0.054)		(0.056)	
H_MRI	0.07**	0.83	0.06***	1.64	0.01	1.79
	(0.033)		(0.025)		(0.022)	
RIX	−0.01	0.81	−0.02	1.69	−0.08	1.94
	(0.037)		(0.041)		(0.069)	
TLM	0.04	5.56	−0.05	11.15	−0.14	10.62
	(0.089)		(0.123)		(0.219)	
Group B - Stock-Return-Based Measures						
BT11P	0.02	1.01	0.06**	2.85	0.08**	2.89
	(0.042)		(0.029)		(0.043)	
CJI	0.05	1.79	0.03	3.29	0.04	3.31
	(0.053)		(0.054)		(0.075)	
JumpRisk	0.07***	1.97	0.08***	4.24	0.11***	5.78
	(0.017)		(0.025)		(0.034)	
JumpRP	−0.15**	3.36	−0.17**	6.71	−0.01	6.72
	(0.084)		(0.072)		(0.081)	
λ_{Hill}	0.03	0.45	0.02	0.97	0.02	1.08
	(0.024)		(0.021)		(0.027)	
Group C - Option-Return-Based Measures						
ADBear	0.01	0.47	−0.01	0.93	0.08***	1.45
	(0.017)		(0.019)		(0.040)	
JUMP	0.00	0.04	0.01*	0.15	0.02	0.10
	(0.017)		(0.009)		(0.017)	
Group D - Macroeconomic Measures						
LE	0.07**	2.76	0.10**	6.25	0.12**	7.58
	(0.034)		(0.040)		(0.057)	
PCOneAll	0.35***	6.67	0.36***	13.53	0.26**	12.91
	(0.112)		(0.106)		(0.145)	
PCOneOption	0.24***	6.49	0.19**	12.74	−0.10	11.32
	(0.111)		(0.100)		(0.205)	
PCOneStReturn	0.00	4.07	0.04	8.76	0.15*	9.64
	(0.051)		(0.048)		(0.106)	
PCOneOpReturn	0.01	0.32	0.00	0.73	0.06***	0.94
	(0.020)		(0.015)		(0.026)	
Controls	*Yes*		*Yes*		*Yes*	

Table A13 Predictability of Left Tail Variation: Block Bootstrap

This table presents the coefficients from a predictive regression for future left tail variation. We perform single regressions of the standardized realized left tail variation on each lagged tail risk measure:

$$LTV^{\mathbb{P}}_{t+\Delta t} = a + b \cdot TRM_t + c \cdot LTV^{\mathbb{P}}_t + d \cdot VIX_t + \epsilon_{t+\Delta t}.$$

TRM_t is the current observation of a tail risk measure. We control for the lagged left tail variation $LTV^{\mathbb{P}}_t$ and the current level of the VIX (VIX_t). We use three different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), and (iii) one-month (*Monthly*). In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the block bootstrap of Lahiri (1999). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. "*PCOneAll*", "*PCOneOption*", "*PCOneStReturn*", and "*PCOneOpReturn*" denote the first PCs of all measures, option-implied, stock-return-based, and option-return-based tail risk measures, respectively. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2
Group A - Option-Implied Measures						
BT11Q	0.19^{**}	2.88	0.30^{**}	8.52	0.14	9.27
	(0.098)		(0.173)		(0.140)	
BT14Q	0.10^{*}	2.18	0.18^{*}	6.67	0.09	6.05
	(0.070)		(0.133)		(0.083)	
BTX15prob	-0.12^{**}	0.96	-0.23^{*}	2.99	-0.23^{*}	3.40
	(0.057)		(0.152)		(0.183)	
BTX15Q	0.09^{*}	1.95	0.13	5.42	0.02	4.77
	(0.053)		(0.111)		(0.093)	
H_MRI	0.03^{**}	0.31	0.05^{*}	0.97	-0.01	1.35
	(0.020)		(0.050)		(0.041)	
RIX	-0.03	0.19	-0.06	0.60	-0.06	0.86
	(0.028)		(0.062)		(0.081)	
TLM	0.05	1.99	0.14	6.39	-0.07	6.73
	(0.088)		(0.219)		(0.271)	
Group B - Stock-Return-Based Measures						
BT11P	-0.01	0.23	0.04^{*}	1.30	0.06^{**}	1.91
	(0.046)		(0.029)		(0.034)	
CJI	0.02	0.62	0.04	1.84	0.03	2.30
	(0.030)		(0.053)		(0.055)	
JumpRisk	0.03^{**}	0.63	0.06^{**}	1.95	0.09^{**}	3.50
	(0.014)		(0.030)		(0.076)	
JumpRP	-0.09^{*}	1.18	-0.15	3.64	-0.03	4.18
	(0.053)		(0.164)		(0.134)	
λ_{Hill}	0.01	0.21	0.01	0.68	0.00	1.03
	(0.018)		(0.037)		(0.050)	
Group C - Option-Return-Based Measures						
ADBear	0.01	0.22	0.02	0.64	0.06^{**}	1.02
	(0.026)		(0.038)		(0.049)	
JUMP	0.04^{**}	0.24	0.02^{*}	0.10	0.03^{***}	0.16
	(0.031)		(0.014)		(0.021)	
Group D - Macroeconomic Measures						
LE	0.03^{*}	0.89	0.06^{**}	2.81	0.14^{***}	5.83
	(0.024)		(0.041)		(0.063)	
PCOneAll	0.20^{**}	2.30	0.32^{**}	7.12	0.29^{**}	8.47
	(0.099)		(0.145)		(0.191)	
PCOneOption	0.14	2.27	0.19	6.78	-0.05	7.08
	(0.114)		(0.223)		(0.301)	
PCOneStReturn	-0.02	1.37	0.02	4.66	0.13	6.47
	(0.064)		(0.111)		(0.119)	
PCOneOpReturn	0.04^{*}	0.35	0.02	0.51	0.06^{**}	0.81
	(0.031)		(0.032)		(0.040)	
Controls	*Yes*		*Yes*		*Yes*	

Table A14 Multiple Predictability of Left Tail Variation: Block Bootstrap

This table presents the coefficients from a predictive regression for future left tail variation. We perform multiple regressions of the realized left tail variation on the lagged tail risk measures:

$$LTV^{\mathbb{P}}_{t+\Delta t} = a + b \cdot TRM_t + c \cdot LTV^{\mathbb{P}}_t + d \cdot VIX_t + \epsilon_{t+\Delta t}.$$

TRM_t is a vector of the current observations of the tail risk measures. We control for the lagged left tail variation $LTV^{\mathbb{P}}_t$ and the current level of the VIX (VIX_t). We use three different forecast horizons Δt: (i) one-day ($Daily$), (ii) one-week ($Weekly$), and (iii) one-month ($Monthly$). For each forecast horizon, we first perform variable selection based on the PcGets algorithm. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with 29 lags. Statistical inference is based on the block bootstrap of Lahiri (1999). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	$Daily$	R^2	$Weekly$	R^2	$Monthly$	R^2
Group A - Option-Implied Measures						
$BT11Q$	0.19^{***}	4.15			-0.02^{***}	10.27
	(0.046)				(0.008)	
$BT14Q$						
$BTX15prob$						
$BTX15Q$						
H_MRI						
RIX						
TLM						
Group B - Stock-Return-Based Measures						
$BT11P$						
CJI						
$JumpRisk$	0.04^{***}	0.65				
	(0.015)					
$JumpRP$						
λ_{Hill}						
Group C - Option-Return-Based Measures						
$ADBear$						
$JUMP$						
Group D - Macroeconomic Measures						
LE			0.04^{***}	3.12	0.01^{***}	4.76
			(0.010)		(0.004)	
$Controls$	Yes		Yes		Yes	

Table A15 Return Predictability: Block Bootstrap

This table presents the coefficients from a return predictability regression. We perform single regressions of the market excess returns on each lagged tail risk measure:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot Controls_t + \epsilon_{t+\Delta t}.$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is the current observation of a tail risk measure. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), (iii) one-month (*Monthly*), and (iv) one-year (*Annually*). In parentheses, we present robust Newey and West (1987) standard errors with lag length chosen to be the maximum of 29 and the number of overlapping observations. Statistical inference is based on the block bootstrap of Lahiri (1999). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. "*PCOneAll*", "*PCOneOption*", "*PCOneStReturn*", and "*PCOneOpReturn*" denote the first PCs of all measures, option-implied, stock-return-based, and option-return-based tail risk measures, respectively. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2	*Annually*	R^2
Group A - Option-Implied Measures								
BT11Q	35.96^{***}	0.52	15.21^{**}	0.51	6.64^{*}	0.48	2.85	2.17
	(7.350)		(5.613)		(4.504)		(1.915)	
BT14Q	5.15	0.02	0.70	0.02	−4.18	0.34	0.71	0.56
	(6.213)		(4.386)		(2.742)		(1.713)	
BTX15prob	11.67^{**}	0.07	7.40	0.24	8.41^{*}	1.18	−2.13	0.49
	(6.121)		(5.518)		(4.242)		(3.193)	
BTX15Q	1.50	0.01	−3.19	0.03	−3.37	0.14	1.14	1.61
	(5.816)		(5.227)		(3.141)		(2.337)	
H_MRI	-7.23^{**}	0.02	−2.98	0.04	−0.65	0.09	3.10	1.01
	(4.028)		(3.313)		(2.917)		(2.620)	
RIX	2.48	0.02	3.16	0.13	3.35	0.61	0.25	1.32
	(5.927)		(5.402)		(4.528)		(3.315)	
TLM	26.51^{***}	0.26	13.48^{**}	0.50	4.11	0.50	−0.73	0.77
	(8.577)		(6.277)		(4.563)		(2.825)	
Group B - Stock-Return-Based Measures								
BT11P	25.95^{***}	0.63	13.89^{***}	0.96	3.91^{***}	0.29	0.58	0.10
	(5.859)		(2.855)		(1.383)		(0.394)	
CJI	3.75	0.01	1.49	0.03	−0.03	0.05	1.22	0.70
	(4.335)		(4.330)		(2.941)		(1.285)	
JumpRisk	−5.67	0.01	-7.73^{*}	0.05	-10.91^{**}	0.55	-15.36^{***}	12.30
	(5.754)		(5.798)		(5.197)		(2.654)	
JumpRP	15.94^{***}	0.11	11.29^{***}	0.38	4.72	0.52	−2.14	0.46
	(5.360)		(4.721)		(3.997)		(2.319)	
λ_{Hill}	−0.42	0.00	1.36	0.05	0.04	0.08	6.25^{***}	9.38
	(4.057)		(4.015)		(3.469)		(1.610)	
Group C - Option-Return-Based Measures								
ADBear	19.16^{***}	0.39	13.31^{***}	1.07	3.30^{**}	0.30	−0.10	0.01
	(4.820)		(3.073)		(1.565)		(0.381)	
JUMP	2.83	0.01	5.80^{***}	0.20	1.14^{*}	0.03	0.20	0.01
	(6.084)		(1.585)		(0.703)		(0.141)	
Group D - Macroeconomic Measures								
LE	2.20	0.01	0.45	0.03	3.14	0.09	−2.27	0.49
	(7.517)		(7.774)		(5.657)		(2.816)	
PCOneAll	34.45^{***}	0.29	18.66^{***}	0.54	8.19	0.68	1.18	1.38
	(8.736)		(6.265)		(4.955)		(2.975)	
PCOneOption	22.46^{**}	0.18	9.16	0.26	2.74	0.33	0.52	1.21
	(8.518)		(6.161)		(4.788)		(3.112)	
PCOneStReturn	24.42^{***}	0.25	14.23^{***}	0.49	5.39^{*}	0.41	−2.27	0.79
	(5.310)		(4.605)		(3.644)		(1.983)	
PCOneOpReturn	14.06^{***}	0.21	12.22^{***}	0.89	2.84^{**}	0.22	0.06	0.00
	(5.477)		(2.523)		(1.294)		(0.271)	
Controls	*Yes*		*Yes*		*Yes*		*Yes*	

Table A16 Multiple Return Predictability: Block Bootstrap

This table presents the coefficients from a return predictability regression. We perform multiple regressions of the market excess returns on lagged tail risk measures:

$$R_{t+\Delta t} = a + b \cdot TRM_t + c \cdot Controls_t + \epsilon_{t+\Delta t}.$$

$R_{t+\Delta t}$ is the excess return over the period Δt. TRM_t is a vector of the current observations of the tail risk measures. We use the following control variables (in $Controls_t$): variance risk premium, log dividend-price ratio, stochastically detrended risk free rate, consumption–wealth ratio, default spread, and term spread. We use four different forecast horizons Δt: (i) one-day (*Daily*), (ii) one-week (*Weekly*), (iii) one-month (*Monthly*), and (iv) one-year (*Annually*). For each forecast horizon, we first perform variable selection based on the PcGets selection algorithm. Space left blank implies that a measure has not been chosen. In parentheses, we present robust Newey and West (1987) standard errors with lag length chosen to be the maximum of 29 and the number of overlapping observations. Statistical inference is based on the block bootstrap of Lahiri (1999). The columns R^2 present the Lindeman et al. (1980) partial R^2 of each tail risk measure, multiplied by 100. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Daily	R^2	*Weekly*	R^2	*Monthly*	R^2	*Annually*	R^2
Group A - Option-Implied Measures								
BT11Q	47.61^{***}	0.53			10.47^{*}	0.57	6.35^{***}	6.42
	(10.620)				(5.865)		(0.831)	
BT14Q					-6.45^{***}	0.63		
					(2.499)			
BTX15prob					8.32^{*}	1.31		
					(4.793)			
BTX15Q	-21.81^{***}	0.12	-21.65^{***}	0.44	-8.22^{***}	0.36		
	(8.673)		(6.785)		(3.136)			
H_MRI								
RIX								
TLM			22.23^{***}	0.53				
			(7.645)					
Group B - Stock-Return-Based Measures								
BT11P	17.39^{***}	0.46	7.35^{***}	0.51				
	(5.906)		(2.697)					
CJI								
JumpRisk							-13.26^{***}	7.93
							(3.057)	
JumpRP								
λ_{Hill}								
Group C - Option-Return-Based Measures								
ADBear			8.78^{***}	0.70				
			(3.063)					
JUMP								
Group D - Macroeconomic Measures								
LE								
Controls	*Yes*		*Yes*		*Yes*		*Yes*	